Recommended Practice on the Rheology and Hydraulics of Oil-Well Drilling Fluids

API RECOMMENDED PRACTICE 13D
THIRD EDITION, JUNE 1, 1995

American Petroleum Institute
1220 L Street, Northwest
Washington, D.C. 20005

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Exploration and Production Department

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CONTENTS

Page

FOREWORD v

1 SCOPE 1

2 REFERENCES 1
2.1 Standards 1
2.2 Other References 1

3 SYMBOLS 2

4 BASIC CONCEPTS 3
4.1 Flow Regimes 3
4.2 Viscosity 3
4.3 Shear Stress 4
4.4 Shear Rate 4
4.5 Relationship of Shear Stress and Shear Rate 5

5 TYPES OF FLUIDS 5
5.1 Description 5
5.2 Newtonian Fluids 5
5.3 Non-Newtonian Fluids 5
5.4 Rheological Models 6

6 EQUIPMENT FOR MEASUREMENT OF RHEOLOGICAL PROPERTIES 7
6.1 Orifice Viscometer—Marsh Funnel 7
6.2 Concentric Cylinder Viscometer 7
6.3 Telescopic-Shear Viscometer—Fann Model 5STDL Consistometer 15
6.4 Pipe Viscometer 16
6.5 Portable Capillary Viscometer 16

7 DATA ANALYSIS 17
7.1 Description 17
7.2 Rheological Profile Curves 17
7.3 Mathematical Flow Models 17
7.4 Analysis of Bingham Plastic Model Data 19
7.5 Mathematical Analysis of Power Law Data 19
7.6 Effects of Temperature and Pressure on Viscosity 20

8 APPLICATION OF RHEOLOGICAL DATA 21
8.1 Description 21
8.2 Friction Loss in Pipe 22
8.3 Friction Loss in an Annulus 22
8.4 Friction Loss in Bit Nozzles 22
8.5 Hydrostatic Pressure Gradient 23
8.6 Circulating Pressure Gradient 23
8.7 Equivalent Circulating Density 23
8.8 Standpipe Pressure 23

9 SETTLING VELOCITY OF DRILL CUTTINGS 23
9.1 Description 23
9.2 Settling of Particles in Water 23
9.3 Estimation of Settling Velocity 24

APPENDIX A—RHEOLOGICAL EXAMPLE CALCULATIONS 25
APPENDIX B—SETTLING VELOCITY EXAMPLE CALCULATIONS 29

Figures

1—Parallel Plates Showing Shear Rate in Fluid-Filled Gap as One Plate Slides Past Another ... 4
2—Concentric Cylinder Viscometer ... 7
3—Fann 34A ... 9
4—Fann HC34A ... 9
5—Baroid 280 ... 9
6—Fann 35A ... 9
7—Fann 39B ... 10
8—OFI 800 ... 10
9—Chan 35 ... 10
10—Baroid 286 ... 11
11—Fann 35A/SR112 ... 11
12—Haake VT500 ... 11
13—Haake RV20 ... 12
14—Fann 50C ... 13
15—Fann 70 ... 14
16—Chandler 7400 ... 14
17—Haake RV20/D100 ... 14
18—Huxley Burtram HTHP Viscometer ... 15
19—Fann 55TDL Consistometer ... 16
20—Linear Shear Stress—Shear Rate Plots ... 17
21—Log-Log Effective Viscosity—Shear Rate Plots ... 17
22—Downhole Viscosity Correction Factor Water Base Mud ... 21
23—Downhole Viscosity Correction Factor Oil Muds Containing Asphalt ... 21
24—Downhole Viscosity Correction Factor Oil Muds Containing Oil-Wet Inorganic Viscosifiers ... 21
25—Settling Velocity of Drill Cuttings in Water ... 24

Tables

1—Low Temperature, Nonpressurized Concentric Cylinder Viscometers ... 8
2—High Temperature, Pressurized Concentric Cylinder Viscometers ... 15
3—Equivalent Diameters of Irregularly Shaped Particles ... 23

FOREWORD

This Recommended Practice is under the jurisdiction of the API Committee on Standardization of Drilling Fluid Materials. This edition of Recommended Practice 13D includes revisions adopted at the 1993 Standardization Conference and subsequently approved by letter ballot as reported in Circ PS-2020.

This standard shall become effective on the date printed on the cover but may be used voluntarily from the date of distribution.

Recommended Practice on the Rheology and Hydraulics of Oil-Well Drilling Fluids

1 Scope

1.1 The objective of this Recommended Practice is to provide a basic understanding of and guidance about drilling fluid rheology and hydraulics and their application to drilling operations.

1.2 Rheology is the study of the deformation and flow of matter. Drilling fluid hydraulics pertains to both laminar and turbulent flow regimes.

1.3 For this Recommended Practice, rheology is the study of the flow characteristics of a drilling fluid and how these characteristics affect movement of the fluid. Specific measurements are made on a fluid to determine rheological parameters of a fluid under a variety of conditions. From this information the circulating system can be designed or evaluated regarding how it will accomplish certain desired objectives. Drilling fluid rheology is important in the following determinations:

a. Calculating friction loss in pipe or annulus.
b. Calculating swab and surge pressure when running or pulling pipe.
c. Determining the equivalent circulating density of the drilling fluid.
d. Determining the flow profile in the annulus.
e. Estimating hole cleaning efficiency.
f. Evaluating fluid suspension capacity.
g. Determining the jet nozzle velocity and friction loss at the bit.
h. Determining the settling velocity of drill cuttings in vertical holes.

1.4 The discussion of rheology in this Recommended Practice is limited to single phase liquid flow. Some commonly used concepts pertinent to rheology and flow are presented. Mathematical models relating shear stress to shear rate and formulas for estimating pressure drops, equivalent circulating densities and settling velocities of drill cuttings are included.[1]

1.5 The conventional English unit system is used in this Recommended Practice.

1.6 Conversion factors and examples are included for all calculations so that English units can be readily converted to metric (SI) units.[2]

1.7 Where units are not specified, as in the development of equations, any consistent system of units may be used.

1.8 The concepts of viscosity, shear stress, and shear rate are very important in understanding the flow characteristics of a fluid. The measurement of these properties allows a mathematical description of circulating fluid flow. The rheological properties of a drilling fluid directly affect its flow characteristics and all hydraulic calculations. They must be controlled for the fluid to perform its various functions.

2 References

2.1 STANDARDS

Unless otherwise specified, the most recent editions or revisions of the following standards shall, to the extent specified herein, form a part of this Recommended Practice.

API

1. RP 13B-1 Recommended Practice Standard Procedure for Field Testing Water-Based Drilling Fluids.
2. RP 13B-2 Recommended Practice Standard Procedure for Field Testing Oil-Based Drilling Fluids.
3. PUBL 2564 Conversion of Operational and Process Measurements Units to the Metric (SI) System.

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[1]Refer to Reference 13.
[2]Refer to Reference 3.

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3 Symbols

For the purposes of this Recommended Practice, the following definitions of symbols apply:

A = Surface area
D = Diameter
D_n = Bit nozzle diameter
D_p = Equivalent particle diameter
D_1 = Inner annulus diameter
D_2 = Outer annulus diameter
F = Force
G = Gravity constant
K = Fluid consistency index
K_a = Fluid consistency index in annulus
K_p = Fluid consistency index in pipe
K_s = Fluid consistency index in settling
L = Length
L_m = Measured depth
L_v = True vertical depth
N_{Re} = Reynolds number
N_{Rea} = Reynolds number in annulus
N_{Rep} = Reynolds number in pipe
P = Pressure
P_a = Pressure drop in annulus
P_c = Circulating pressure
P_h = Hydrostatic pressure
P_n = Pressure drop in bit nozzles
P_p = Pressure drop in pipe
PV = Plastic viscosity (PV = η)
Q = Volumetric flow rate
R = Fann Viscometer reading
R_3 = Fann Viscometer reading at 3 rpm
R_{100} = Fann Viscometer reading at 100 rpm
R_{300} = Fann Viscometer reading at 300 rpm
R_{600} = Fann Viscometer reading at 600 rpm
T = Temperature
V = Velocity
V_a = Average bulk velocity in annulus
V_p = Average bulk velocity in pipe
V_s = Average settling velocity
V_o = Volume of settling particle
YP = Yield point
a = Friction factor constant
b = Friction factor exponent
f = Friction factor
f_a = Friction factor in annulus
f_p = Friction factor in pipe
n = Power law exponent
n_a = Power law exponent in annulus

n_p = Power law exponent in pipe
n_s = Power law exponent in settling
Ψ = Ratio of particle surface areas
α = Pressure constant
β = Temperature constant
γ = Shear rate
γ_s = Settling shear rate
γ_w = Shear rate at wall
γ_{wa} = Shear rate at annulus wall
γ_{wp} = Shear rate at pipe wall
η = Plastic viscosity (η = PV)
θ = Angle
μ = Viscosity
μ_e = Effective viscosity
μ_{ea} = Effective viscosity in annulus
μ_{ep} = Effective viscosity in pipe
μ_{es} = Effective viscosity in settling
ρ = Density of fluid
ρ_c = Equivalent circulating density
ρ_p = Density of a particle
τ = Shear stress
τ_w = Shear stress at wall
τ_{wa} = Shear stress at wall in annulus
τ_{wp} = Shear stress at wall in pipe
τ_y = Yield stress
ω = Angular momentum

4 Basic Concepts

4.1 FLOW REGIMES

4.1.1 The behavior of a fluid is determined by the flow regime, which in turn has a direct effect on the ability of that fluid to perform its basic functions. The flow can be either laminar or turbulent, depending on the fluid velocity, size of the flow channel, fluid density, and viscosity. Between laminar and turbulent flow, the fluid will pass through a transition region where the movement of the fluid has both laminar and turbulent characteristics. It is important to know which of the flow regimes is present in a particular situation to evaluate the performance of a fluid.

4.1.2 In laminar flow, the fluid moves parallel to the walls of the flow channel in smooth lines. Flow tends to be laminar when moving slowly or when the fluid is viscous. In laminar flow, the pressure required to move the fluid increases with increases in the velocity and viscosity.

4.1.3 In turbulent flow, the fluid is swirling and eddying as it moves along the flow channel, even though the bulk of the fluid moves forward. These velocity fluctuations arise spontaneously. Wall roughness or changes in flow direction will increase the amount of turbulence. Flow tends to be turbulent with higher velocities or when the fluid has low viscosity. In turbulent flow, the pressure required to move the fluid increases linearly with density and approximately with the square of the velocity. This means more pump pressure is required to move a fluid in turbulent flow than in laminar flow.

4.1.4 The transition between laminar and turbulent flow is controlled by the relative importance of viscous forces and inertial forces in the flow. In laminar flow, the viscous forces dominate, while in turbulent flow the inertial forces are more important. For Newtonian fluids, viscous forces vary linearly with the flow rate, while the inertial forces vary as the square of the flow rate.[3]

4.1.5 The ratio of inertial forces to viscous forces is the Reynolds Number. If consistent units are chosen, this ratio will be dimensionless and the Reynolds Number (N_{Re}) will be:

$$N_{Re} = \frac{DV\rho}{\mu} \tag{1}$$

Where:

D = dimension of the flow channel
V = average flow velocity
ρ = fluid density
μ = viscosity

4.1.6 The flow of any particular liquid in any particular flow channel can be either laminar, transitional, or turbulent. The transition occurs at a critical velocity. It normally occurs over a range of velocities corresponding to Reynolds Number between 2000 and 4000.

4.2 VISCOSITY

4.2.1 Viscosity is defined as the ratio of shear stress to shear rate. The traditional units of viscosity are dyne-second/centimeter², which is termed poise. Since one poise represents a relatively high viscosity for most fluids, the term centipoise (cP) is normally used. A centipoise is equal to one-hundredth of poise or one millipascal-second.

$$\mu = \frac{\tau}{\gamma} \tag{2}$$

Where:

μ = viscosity
τ = shear stress
γ = shear rate

4.2.2 Most drilling fluid viscosity is not a constant value. It varies with shear rate. To check for rate dependent effects, shear stress measurements are made at a number of shear rates. From these measured data, rheological parameters can be calculated or can be plotted as viscosity versus shear rate.

4.2.3 The term effective viscosity is used to describe the viscosity either measured or calculated at the shear rate cor-

[3]Refer to References 7, 8 and 25.

responding to existing flow conditions in the wellbore or drill pipe. This special term is created to differentiate the viscosity as discussed in this section from other viscosity terms. To be meaningful a viscosity measurement must always specify the shear rate.

4.3 SHEAR STRESS

4.3.1 Shear stress is the force required to sustain a particular rate of fluid flow and is measured as a force per unit area. Suppose, in the parallel-plate example (see Fig. 1), that a force of 1.0 dyne is applied to each square centimeter of the top plate to keep it moving. Then the shear stress would be 1.0 dyne/cm². The same force in the opposite direction is needed on the bottom plate to keep it from moving. The same shear stress of 1.0 dyne/cm² is found at any level in the fluid.

4.3.2 Shear stress (τ) is expressed mathematically as:

$$\tau = \frac{F}{A} \tag{3}$$

Where:

F = force
A = area of the surface subject to stress

4.3.3 In a pipe, the force pushing a column of liquid through the pipe is expressed as the pressure on the end of the liquid column times the area of the end of the column:

$$F = P\,\frac{\pi D^2}{4} \tag{4}$$

Where:

D = diameter of pipe
P = pressure on end of liquid column

4.3.4 The area of the fluid surface in contact with the pipe wall over the length is given by:

$$A = \pi D L \tag{5}$$

Where:

A = area of the fluid surface
L = length

4.3.5 Thus, the shear stress at the pipe wall is expressed as:

$$\tau_w = \frac{F}{A} = \frac{\frac{\pi D^2}{4}}{\pi D L} \tag{6}$$

4.3.6 In an annulus with inner and outer diameters known, the shear stress is expressed in the same manner:

$$F = \frac{P\pi D_2^2}{4} - \frac{P\pi D_1^2}{4} = P\pi\,\frac{D_2^2 - D_1^2}{4} \tag{7}$$

Where:

D_1 = inner diameter of pipe
D_2 = outer diameter of pipe

and

$$A = \pi D_2 L + \pi D_1 L = \pi L\,(D_2 + D_1) \tag{8}$$

so that

$$\tau_w = \frac{F}{A} = \frac{P\frac{\pi}{4}(D_2 - D_1)(D_2 + D_1)}{\pi L\,(D_2 + D_1)} = \frac{P\,(D_2 - D_1)}{4L} \tag{9}$$

4.4 SHEAR RATE

4.4.1 Shear rate is a velocity gradient measured across the diameter of a pipe or annulus. It is the rate at which one layer of fluid is moving past another layer. As an example, consider two large flat plates parallel to each other and one centimeter (cm) apart. The space between the plates is filled with fluid. If the bottom plate is fixed while the top plate slides parallel to it at a constant velocity of 1 cm/sec, the velocities indicated in Fig. 1 are found within the fluid. The fluid layer near the bottom plate is motionless while the fluid layer near the top plate is moving at almost 1 cm/sec. Halfway between the plates the fluid velocity is the average 0.5 cm/sec.

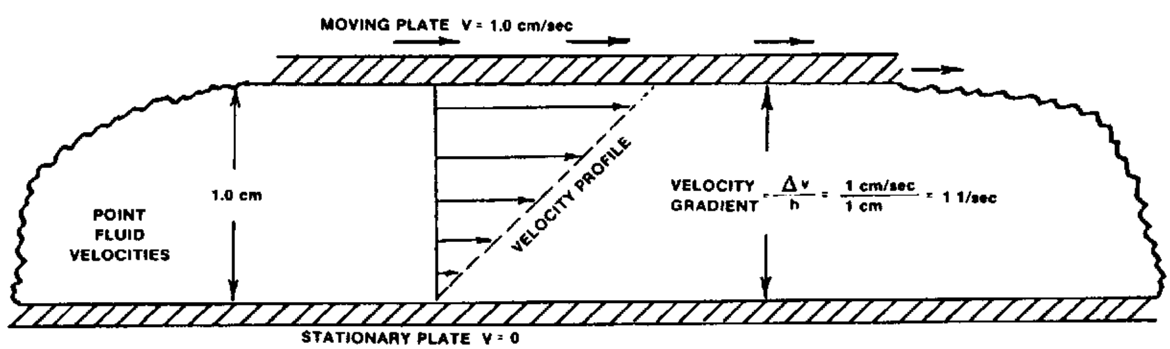

Figure 1—Parallel Plates Showing Shear Rate in Fluid-Filled Gap as One Plate Slides Past Another

4.4.2 The velocity gradient is the rate of change of velocity (ΔV) with distance from the wall (h). For the simple case of Fig. 1, the shear rate is $\Delta V/h$ and will have units of 1/time. The reciprocal second (1/sec or sec^{-1}) is the standard unit of shear rate.

4.4.3 This reference example is unusual in that the shear rate is constant throughout the fluid. This situation is not the case with a circulating fluid. In laminar flow inside a pipe, for example, the shear rate is highest next to the pipe wall. An average shear rate may be used for calculations, but the shear rate itself is not constant across the flow channel.

4.4.4 It is important to express the above concept mathematically so that models and calculations can be developed. Shear rate (γ) is defined as:

$$\gamma = \frac{dV}{dr} \qquad (10)$$

Where:

dV = velocity change between fluid layers
dr = distance between fluid layers

4.4.5 Shear rate at the pipe wall (γ_{wp}) can be expressed as a function of the average velocity (V) and the diameter of the pipe (D).[4]

$$\gamma_{w_p} = f(V,D) = \frac{8V_p}{D} \qquad (11)$$

in which

$$V_p = \frac{Q}{A} = \frac{4Q}{\pi D^2} \qquad (12)$$

Where:

Q = volumetric flow rate
A = cross sectional area
D = pipe diameter

4.4.6 In an annulus of outside diameter (D_2) and inside diameter (D_1), the wall shear rate can be shown to be:[4]

$$\gamma_{w_a} = f(V,D_1,D_2) = \frac{12V_a}{D_2 - D_1} \qquad (13)$$

in which

$$V_a = \frac{4Q}{\pi\,(D_2^2 - D_1^2)} \qquad (14)$$

4.5 RELATIONSHIP OF SHEAR STRESS AND SHEAR RATE

4.5.1 In summary, the shear stress is the force per unit area required to sustain fluid flow. Shear rate is the rate at which the fluid velocity changes with respect to the distance from the wall. Viscosity is the ratio of the shear stress to shear rate. The mathematical relationship between shear rate and shear stress is the rheological model of the fluid.

4.5.2 When a drill cutting particle settles in a drilling fluid, the fluid immediately surrounding the particle is subjected to a shear rate defined as settling shear rate (γ_s):

$$\gamma_s = \frac{12\,V_s}{D_p} \qquad (15)$$

Where:

V_s = average settling velocity (ft/sec)
D_p = equivalent particale diameter (inch)

The settling shear rate is used to calculate the viscosity of fluid experienced by the settling particle.

5 Types of Fluids

5.1 DESCRIPTION

5.1.1 Fluids can be classified by their rheological behavior. Fluids whose viscosity remains constant with changing shear rate are known as Newtonian fluids. Non-Newtonian fluids are those fluids whose viscosity varies with changing shear rate. Some mathematical models used for hydraulic calculations are shown in this section.

5.1.2 Temperature and pressure affect the viscosity of a fluid.[5] Therefore, to properly describe the drilling fluid flow, the test temperature and pressure must be known.

5.2 NEWTONIAN FLUIDS

5.2.1 Those fluids in which shear stress is directly proportional to shear rate are called Newtonian. Water, glycerin, and light oil are examples.

5.2.2 A single viscosity measurement characterizes a Newtonian fluid.

5.3 NON-NEWTONIAN FLUIDS

5.3.1 Most drilling fluids are not Newtonian; the shear stress is not directly proportional to shear rate. Such fluids are called non-Newtonian.[6] Drilling fluids are shear thinning when they have less viscosity at higher shear rates than at lower shear rates.

5.3.2 The distinction between Newtonian and non-Newtonian fluids is illustrated by using the API standard concentric cylinder viscometer.[7] If the 600-rpm dial reading is twice the 300-rpm reading, the fluid exhibits Newtonian flow behavior. If the 600-rpm reading is less than twice the 300-rpm

[4]Refer to References 7 and 30.

[5]Refer to References 5, 6, and 24.
[6]Refer to References 18 and 29.
[7]Refer to Reference 28.

reading, the fluid is non-Newtonian and shear thinning.

5.3.3 One type of shear thinning fluid will begin to flow as soon as any shearing force or pressure, regardless of how slight, is applied. Such fluid is termed pseudoplastic.[8] Increased shear rate causes a progressive decrease in viscosity.

5.3.4 Another type of shear thinning fluid will not flow until a given shear stress is applied. This shear stress is called the yield stress.

5.3.5 There are non-Newtonian fluids which have dilatant behavior. The viscosity of these fluids increases with increasing shear rate. Dilatant behavior of drilling fluids rarely, if ever, occurs.

5.3.6 Fluids can also exhibit time dependent effects. Under a constant shear rate, the viscosity changes with time until an equilibrium is established. Thixotropic fluids experience a decrease in viscosity with time while rheopectic fluids experience an increase in viscosity with time.

5.3.7 Thixotropic fluids can also exhibit a behavior described as gellation or gel strength. The time dependent forces cause an increase in viscosity as the fluid remains static. Sufficient force must be exerted on the fluid to overcome the gel strength to begin flow.

5.3.8 The range of rheological characteristics of drilling fluids can vary from an elastic gelled solid at one extreme, to a purely viscous Newtonian fluid at the other. The circulating fluids have a very complex flow behavior, yet it is still common practice to express the flow properties in simple rheological terms.

5.3.9 General statements regarding drilling fluids are usually subject to exceptions because of the extraordinary complexity of these fluids.[9]

5.4 RHEOLOGICAL MODELS

5.4.1 Rheological models are intended to provide assistance in characterizing fluid flow. No single commonly-used model completely describes rheological characteristics of drilling fluids over their entire shear rate range. A knowledge of rheological models combined with practical experience is necessary to fully understand fluid performance.

5.4.2 Bingham Plastic Model: The most common rheological model used for drilling fluids is the Bingham plastic model. This model describes a fluid in which the shear stress/shear rate ratio is linear once a specific shear stress has been exceeded. Two parameters, plastic viscosity and yield point, are used to describe this model. Because these constants are determined between the specified shear rates of 500 to 1000 sec^{-1}, this model characterizes a fluid in the higher shear rate range.

5.4.3 Power Law Model: The power law model is used to describe the flow of shear thinning or pseudoplastic drilling fluids. This model describes a fluid in which shear stress versus shear rate is a straight line when plotted on a log-log graph. Since the constants, n and K, from this model are determined from data at any two speeds, it more closely represents an actual fluid over a wide range of shear rates.

5.4.4 Herschel-Buckley (Modified Power Law) Model: The modified power law model is used to describe the flow of a pseudoplastic drilling fluid which requires a yield stress to flow. After the yield stress has been exceeded, the shear rate versus shear stress is linear when plotted on a log-log graph. This model has the advantages of the power law model and more nearly describes the flow of a drilling fluid since it also includes a yield value. The Herschel-Buckley model contains three parameters that are more difficult to use in rheological calculations.

5.4.5 Casson Model: This is a two-parameter model which is based on the square root of the yield stress and the viscosity at infinite shear rate. This model is difficult to use and is rarely applied in calculations of drilling fluid hydraulics.

5.4.6 The rheological parameters recorded in an API Drilling Mud Report are plastic viscosity and yield point from the Bingham plastic model.

5.4.7 The mathematical treatment of Bingham plastic and power law models is discussed in Section 7.

5.4.8 The flow characteristics of a drilling fluid are controlled by the viscosity of the base fluid, the continuous phase, and any solid particles, oil, or gases within the fluid, the discontinuous phases, and the flow channel characteristics, and the volumetric flow rate. Any interactions between the continuous and discontinuous phases, either chemical or physical, have a marked effect on the rheological parameters of a drilling fluid. The constants calculated by use of Bingham, Power Law and other models are only indicators that are commonly used to guide fluid conditioning to obtain the desired rheological properties.

[8]Refer to Reference 16.
[9]Refer to References 20, 21, and 31.

6 Equipment for Measurement of Rheological Properties

6.1 ORIFICE VISCOMETER—MARSH FUNNEL

6.1.1 Description

The Marsh funnel has been widely used as a field measuring instrument. The measurement is referred to as the funnel viscosity and is a timed rate of flow, usually recorded in seconds per quart. The instrument is dimensioned so that by following standard procedures the outflow time of one quart of fresh water at 70°F ± 5°F (21°C ± 2°C) is 26 seconds ± 0.5 second.

6.1.2 Limitations

Funnel viscosity is a rapid, simple test that can be made routinely on a particular drilling fluid system. It is, however, a one-point measurement and therefore does not give any information as to why the viscosity may be high or low. No single funnel viscosity measurement can be taken to represent a consistent value for all drilling fluids of the same type or of the same density.

6.1.3 Operating Procedures

Refer to API Recommended Practice 13B-1, Recommended Practice Standard Procedure for Field Testing Water-Based Drilling Fluids, or Recommended Practice 13B-2, Recommended Practice Standard Procedure for "Field Testing Oil-Based Drilling Fluids", Sections entitled "Marsh Funnel".

6.2 CONCENTRIC CYLINDER VISCOMETER

6.2.1 Low Temperature, Nonpressurized Instruments

6.2.1.1 Description

Concentric cylinder viscometers are rotational instruments powered by an electric motor or a hand crank. Fluid is contained in the annular space between two cylinders. The outer sleeve or rotor sleeve is driven at a constant rotational velocity. The rotation of the rotor sleeve in the fluid produces a torque on the inner cylinder or bob. A torsion spring retains the movement. This mechanism is illustrated in Fig. 2. In most cases, a dial attached to the bob indicates displacement of the bob. Instrument constants have been so adjusted that plastic viscosity and yield point are obtained by readings from rotor sleeve speeds of 300 and 600 rpm. Instruments are also available that are not direct reading but use x-y recorders to record the acquired data.

6.2.1.2 Selection of Instruments

Several models of low temperature, unpressured concentric cylinder viscometers are commonly used in testing drilling fluids.[10] They differ in drive, available speeds, methods of readouts and measuring angles. All permit rapid calculation of plastic viscosity and yield point from readings at 300 rpm and 600 rpm. Table 1 shows some of the models available and their operating limits. Illustrations of instruments are found in Fig. 3 through Fig. 13.

[10]Refer to Reference 28.

6.2.1.3 Operating Procedures

Operating procedures for several models of concentric cylinder viscometers are detailed in API Recommended Practice 13B-1 or Recommended Practice 13B-2. Specific operating procedures for those instruments not included in the Recommended Practice 13B-1 or 13B-2 can be obtained from the manufacturer.

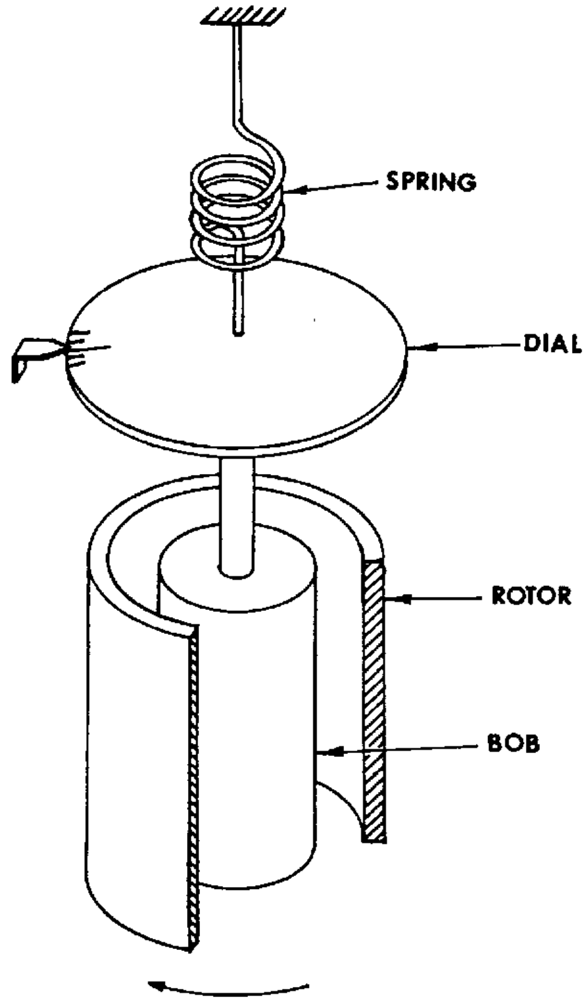

Figure 2—Concentric Cylinder Viscometer

6.2.2 High Temperature, Pressurized Instruments

6.2.2.1 Description

Several instruments are used to measure flow properties of

Table 1—Low Temperature, Nonpressurized Concentric Cylinder Viscometers

Model	Drive	Power	Readout	Rotor Speed rpm	Vis. Range* cP	Max. Temp. °F
Fann 34A	Motor	12V AC-DC	Dial	300, 600 Stir	1-300	200
Fann HC34A	Hand Cranked	—	Dial	300, 600	1-300	200
Baroid 280	Hand Cranked	—	Dial	300, 600 Stir	1-300	200
Fann 35A	Motor	115V, 60 Hz	Dial	3.6,100, 200, 300, 600	1-30,000	200
Fann 35SA	Motor	115V, 50 Hz	Dial	3.6,100, 200, 300, 600	1-30,000	200
Chan 35	Motor	115V, 60 Hz 220V, 50 Hz	Dial	0.9, 2, 3, 6, 10, 20, 30, 60, 100, 200, 300, 600	1-100,000	200
OFI 800	Motor	12V DC 115V, 60 Hz 200V, 50 Hz	Dial	3, 6, 30, 60, 100, 200, 300, 600	1-100,000	200
Fann 35A/SR12	Motor	115V, 60 Hz	Dial	0.9, 1.8, 3.6, 30, 60, 90, 100, 180, 200, 300, 600	1-100,000	200
Fann 35SA/SR12	Motor	115V, 50 Hz	Dial	0.9, 1.8, 3.6, 30, 60, 90, 100, 180, 200, 300, 600	1-100,000	200
Fann 39B	Motor	115V, 50-60 Hz	XY, Yt Recorder	1-600 variable	1-300,000	200
Baroid 286	Motor	12V 115V 220V	Dial	1-625 variable	1-300	200
Haake VT500	Motor	115V, 60 Hz	Digital	0-600 variable	1-100,000	400
Haake RV2	Motor	115V, 60 Hz	Digital	0-1000 variable	1-10,000,000	400

*Newtonian viscosity with standard rotor, bob, spring combination.

drilling fluids at elevated temperatures and pressures. Each instrument has differences in temperature and pressure limitations, and design variation.

a. FANN® Model 50C Viscometer. This instrument (shown in Fig. 14) is designed in the same fashion as the unpressured viscometer. The upper operating limits are 1000 psi and 500°F. Fluid is contained in the annular space between two cylinders with the outer sleeve being driven at a controlled rotational velocity. Torque is exerted on the inner cylinder or bob by the rotation of the outer sleeve in the fluid. This torque is then measured to determine flow properties. Data are recorded either on an x-y recorder or strip chart recorder. This instrument has infinitely variable rotor speeds from 1-625 rpm with a viscosity range of 1-300,000 cP. The temperature range of 0-500° F is programmable. This instrument is also available with an on-board computer for control and data acquisition, as well as a third version that can be interfaced to a personal computer.

b. FANN Model 70 HPHT Viscometer. The high pressure, high temperature instrument (shown in Fig.15) has upper operating limits of 20,000 psi and 500°F. It is a concentric cylinder viscometer that uses the same geometry as the nonpressurized viscometers. Rotor speeds are variable up to 600 rpm. The rotor has external flights to induce circulation. Temperature, pressure, rpm, and shear stress are obtained through digital readout. The digital temperature control has ramp and soak capacities.

c. EG&G CHANDLER ENGINEERING Model 7400. This instrument is a high pressure, high temperature, coaxial cylinder, couette-type rheometer (shown in Fig. 16). Testing limits are 20,000 psi and 400°F. Torque range is 0-540,000 dyne/cm, measured by a precision strain gage sensor. There are 12 evenly spaced preset rotor speeds as well as infinitely variable from 0 to 600 rpm. Temperature is microprocessor controlled. The unit is provided with a data acquisition system that displays temperature, pressure, torque, and rotor speed in real time on a computer monitor. The test fluid is continually circulated in the sample container by the rotor design. The test fluid is separated from the pressurizing medium by a flexible piston to prevent contamination.

d. HAAKE RV20/D100. This instrument (shown in Fig. 17) is a high temperature, pressurized rotational viscometer with upper operating limits of 1400 psi and 572° F. It consists of concentric cylinders mounted in an autoclave. The outer cylinder is bolted to the autoclave top and supports the inner cylinder on a ball bearing. The inner cylinder (or rotor) is connected by a magnetic coupling to a Haake Rotovisco RV20. Computer control is available for automatically plot-

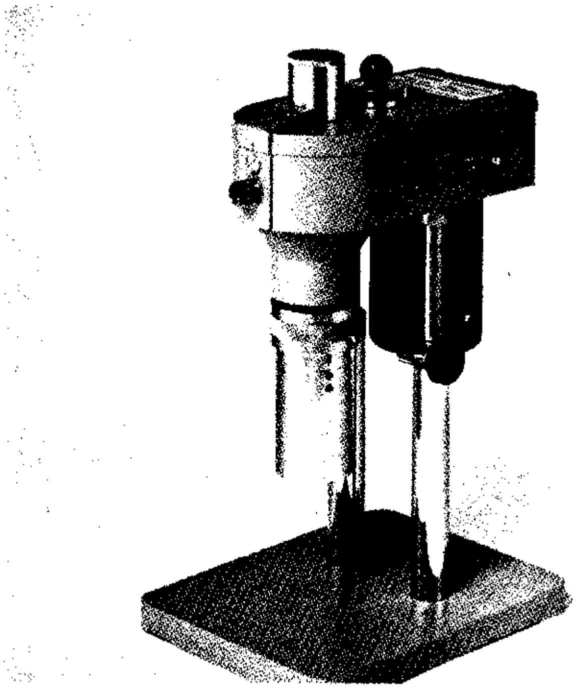

Figure 3—Fann 34A

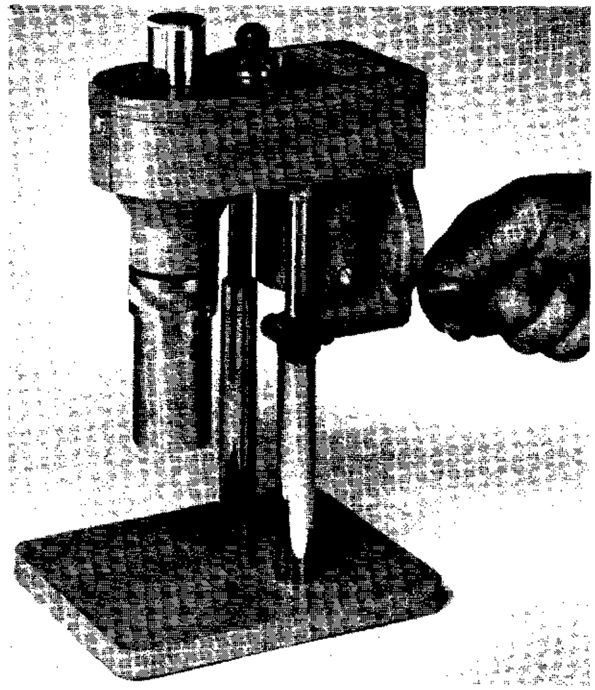

Figure 4—Fann HC34A

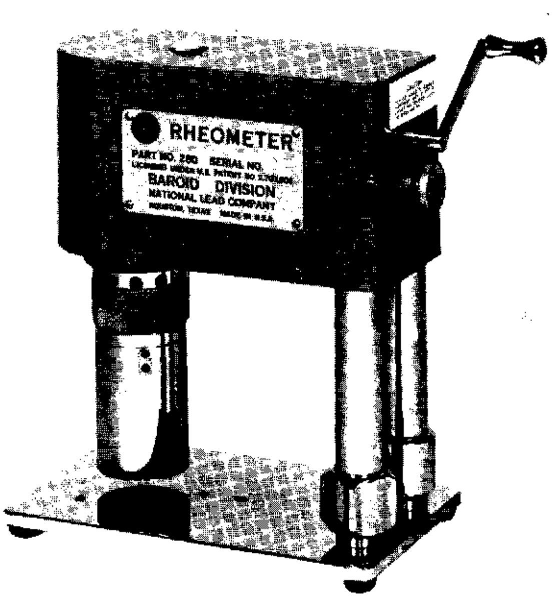

Figure 5—Baroid 280

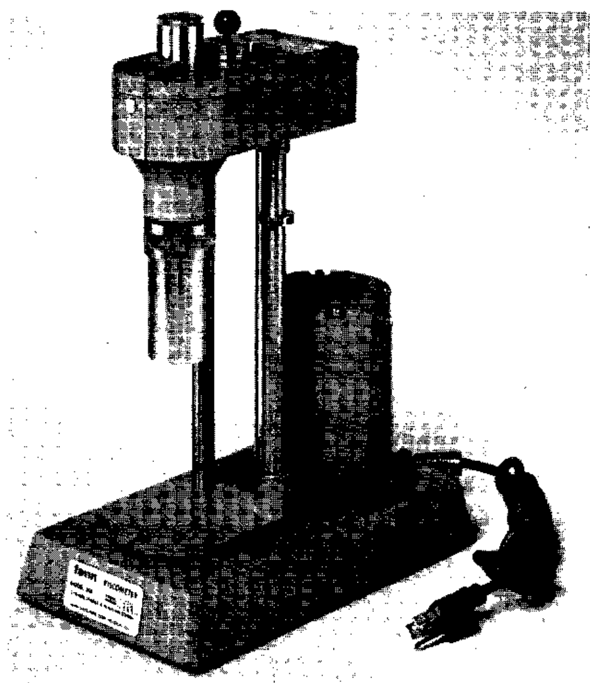

Figure 6—Fann 35A

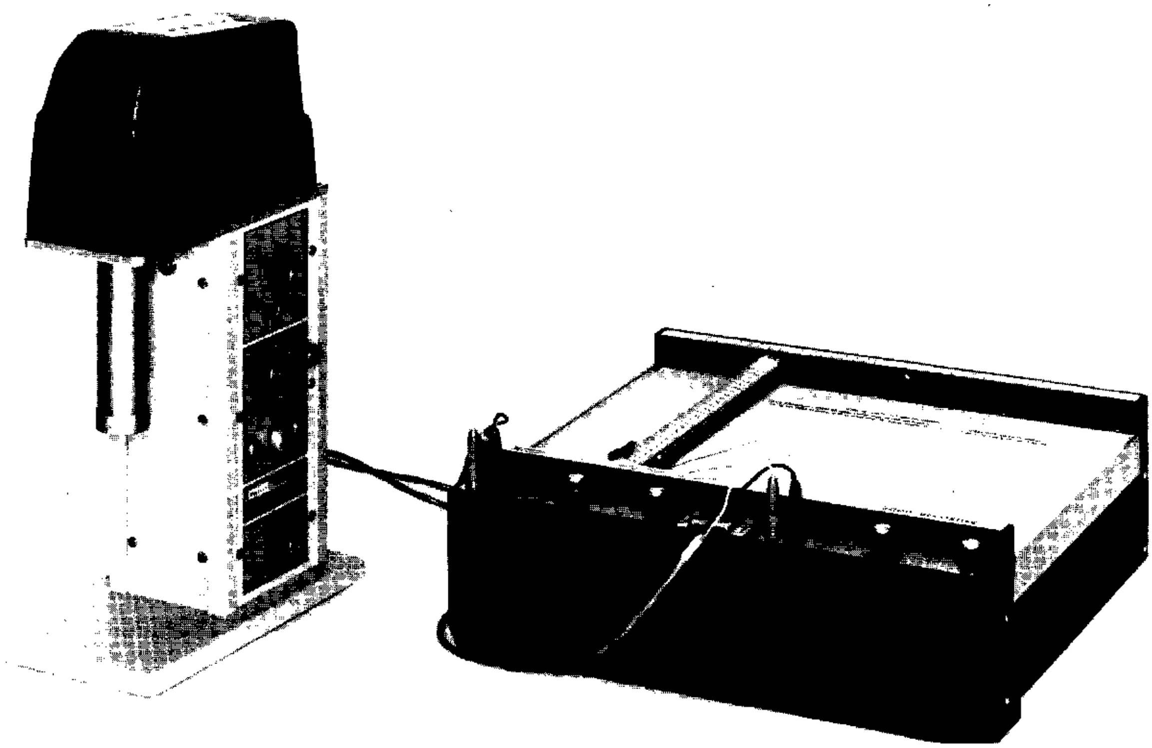

Figure 7—Fann 39B

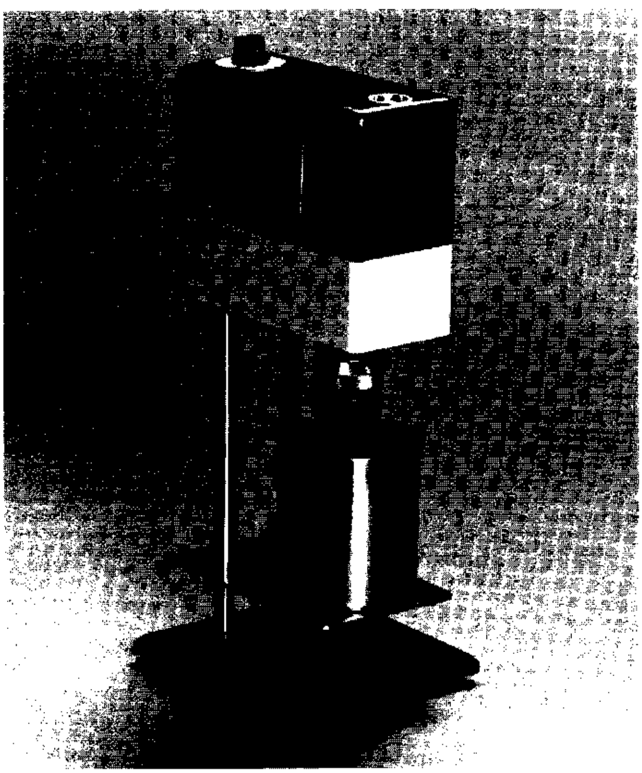

Figure 8—OFI 800

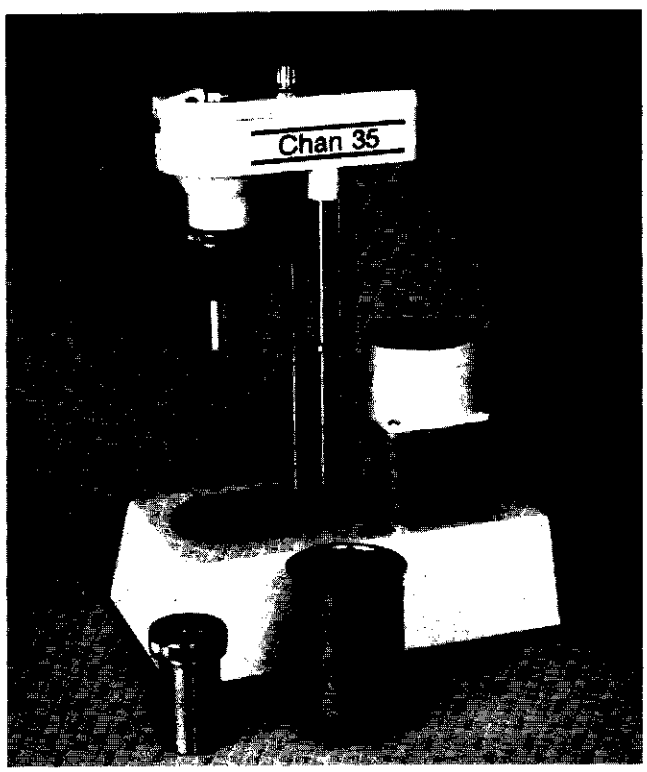

Figure 9—Chan 35

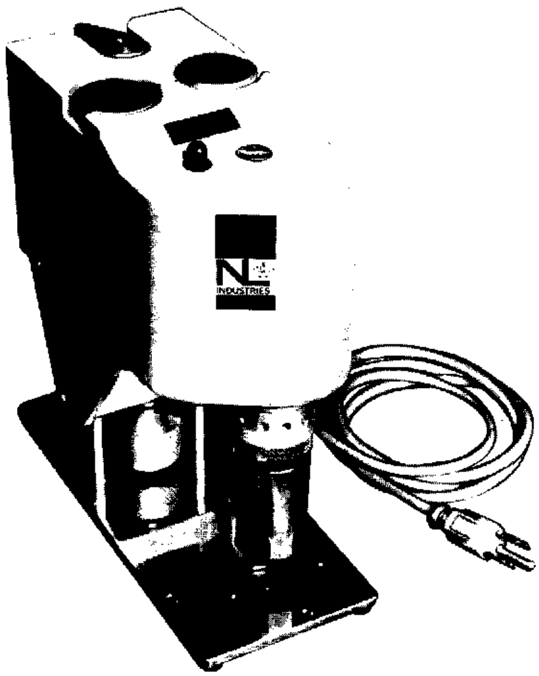

Figure 10—Baroid 286

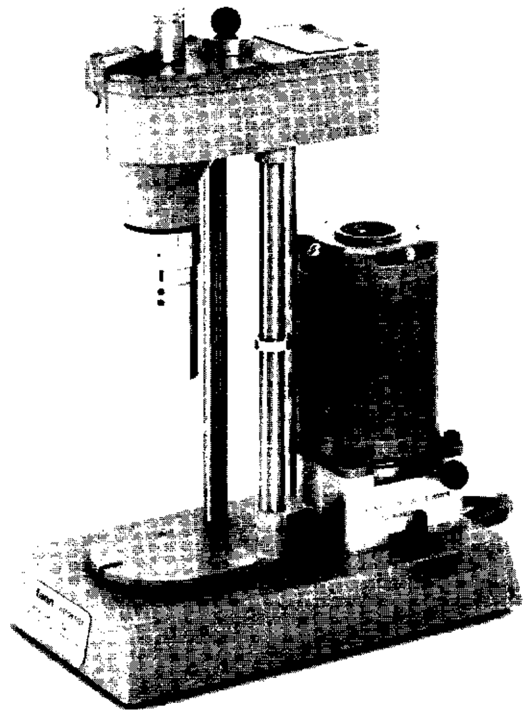

Figure 11—Fann 35A/SR112

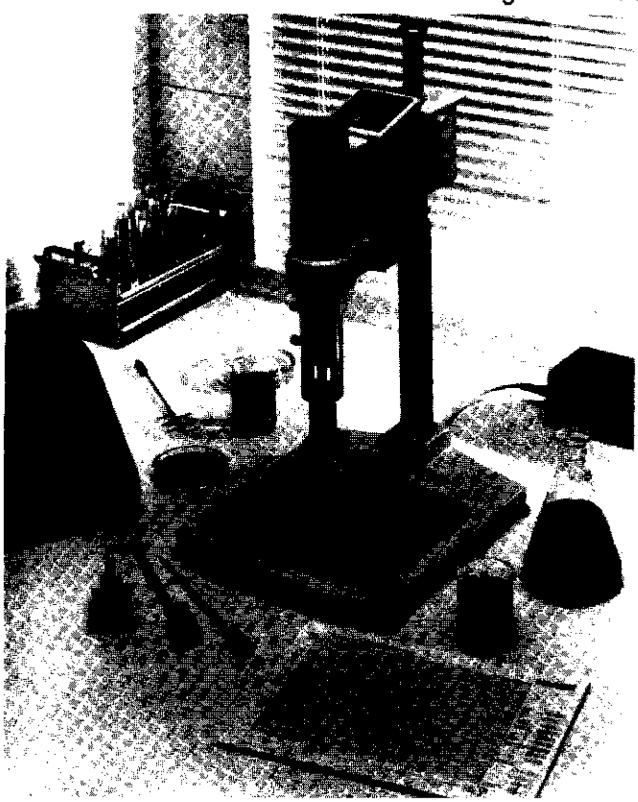

Figure 12—Haake VT500

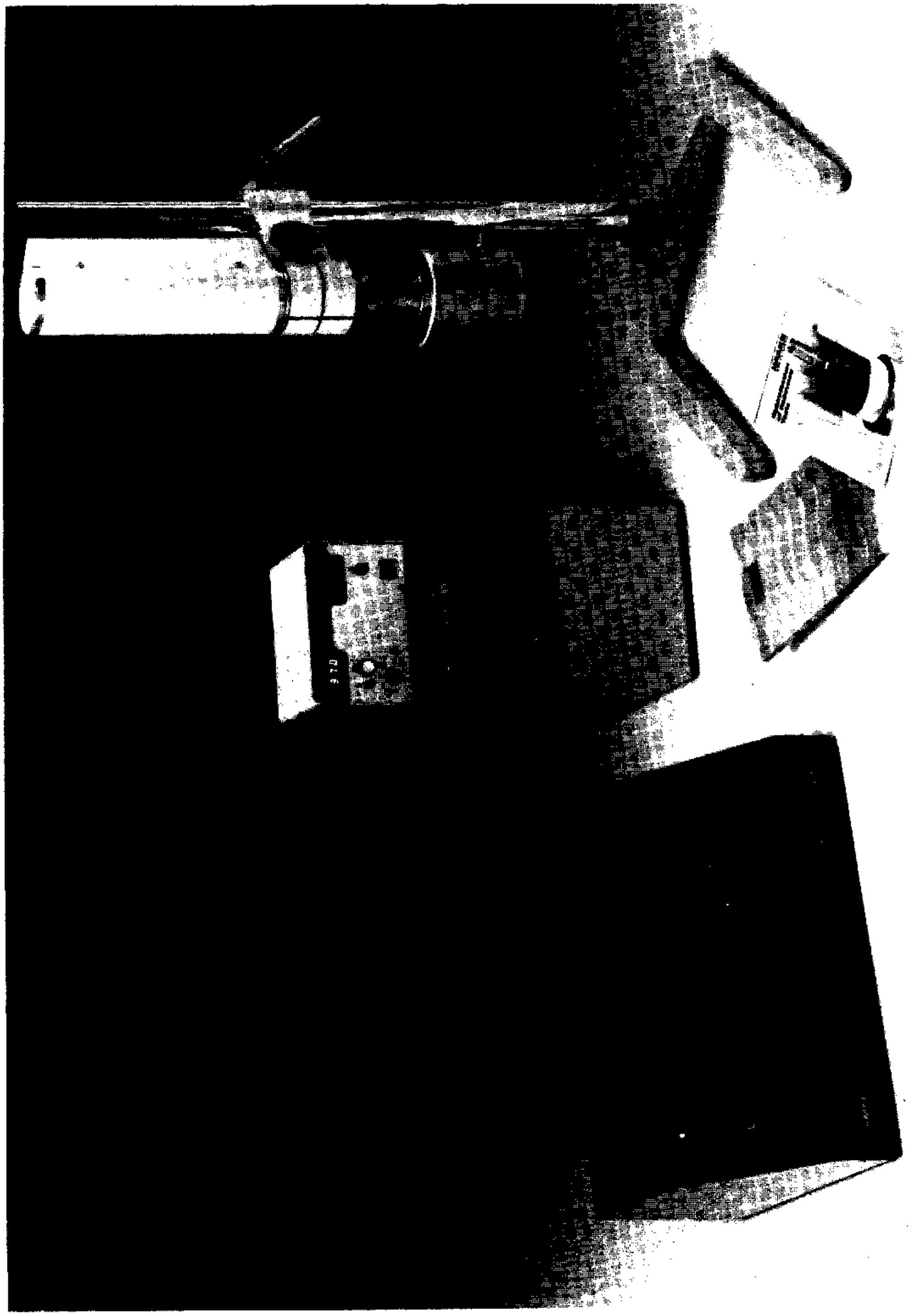

Figure 13—Haake RV20

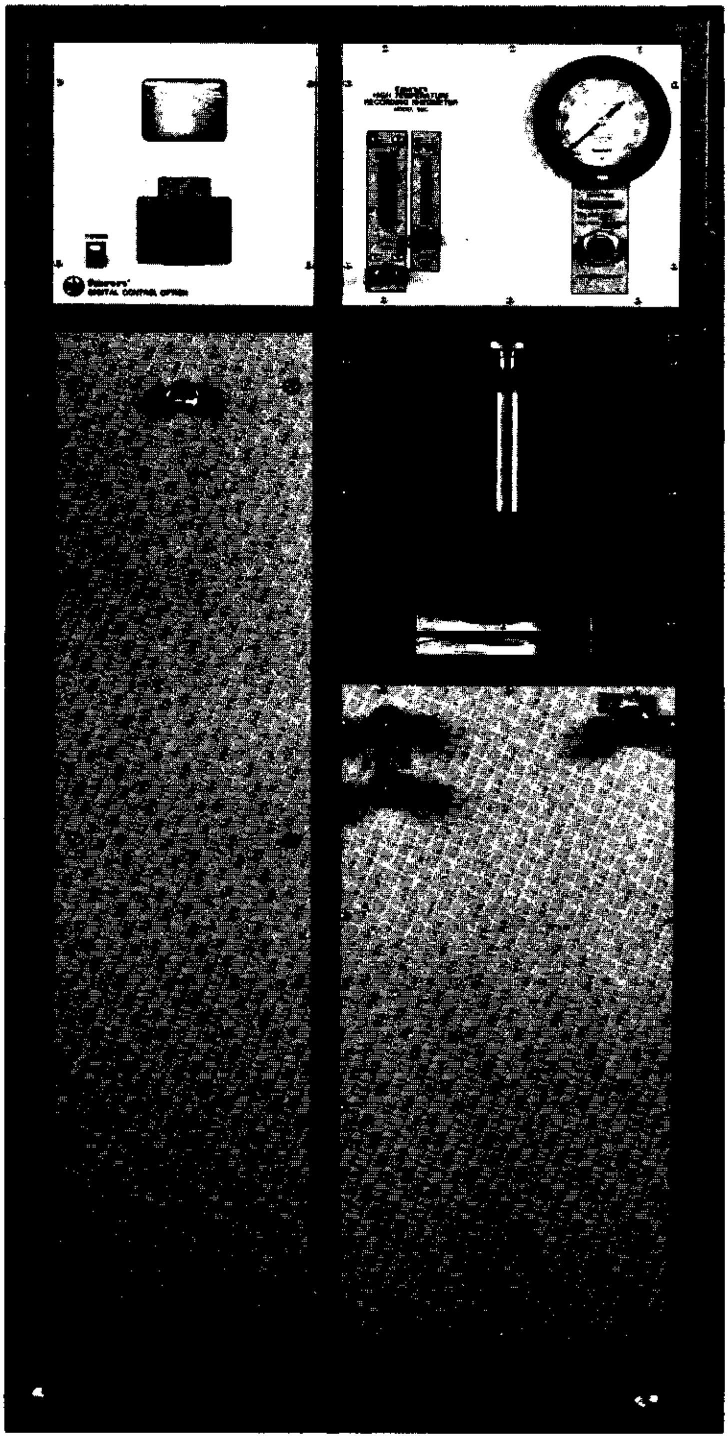

Figure 14—Fann 50C

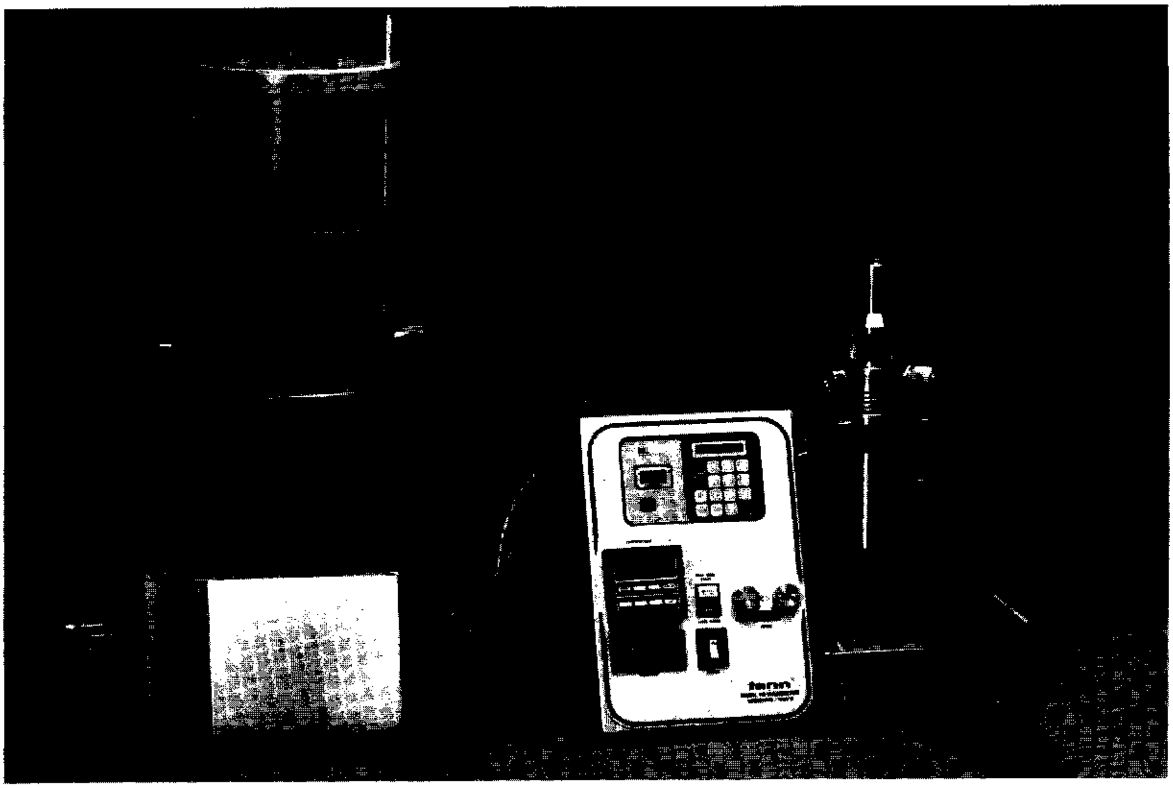

Figure 15—Fann 70

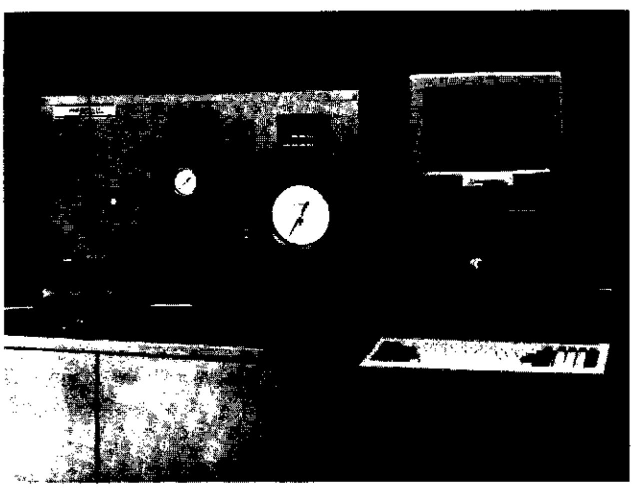

Figure 16—Chandler 7400

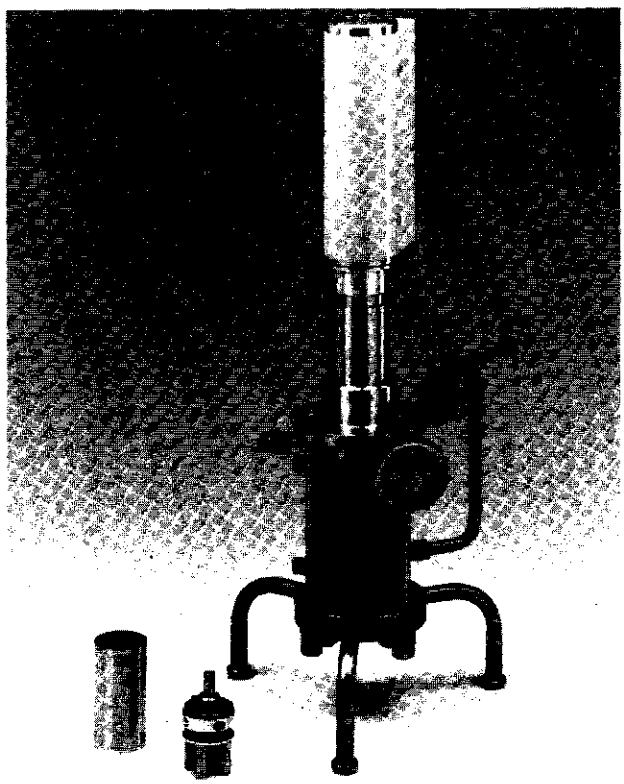

Figure 17—Haake RV20/D100

ting flow curves. The instrument is continuously variable between 0 and 1200 sec-1 and provides automatic data analyses. The torque imparted on the rotor is measured by an electrical torsion bar which provides rapid response. The angular movement of the torsion is a measurement of the transmitted torque; the shear stress is calculated from the torque value by means of an appropriate shear stress constant. A high pressure, high temperature version of this instrument is also available with upper operating limits of 14,000 psi and 662°F. (No photograph of this HPHT equipment is available.)

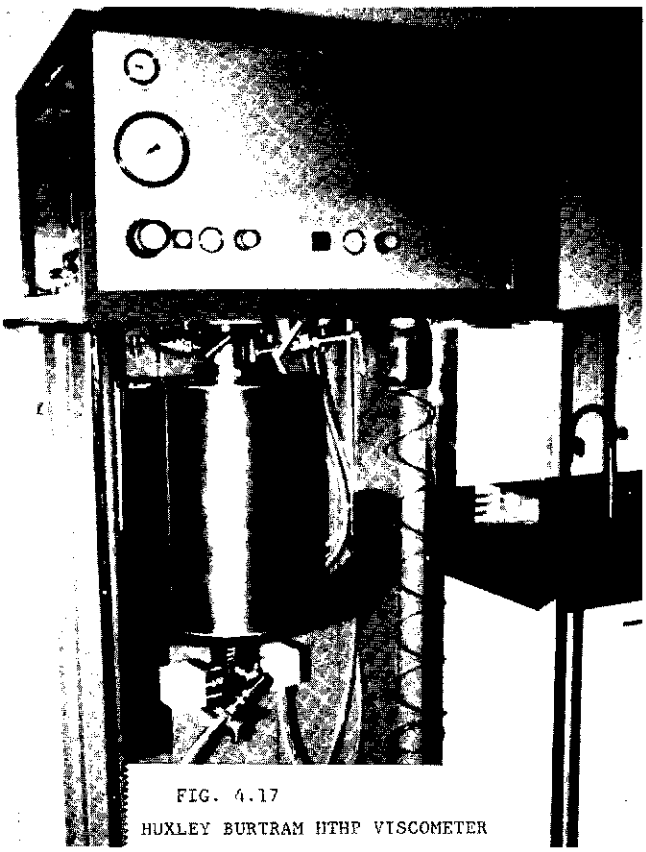

Figure 18—Huxley Burtram HTHP Viscometer

e. HUXLEY-BERTRAM HTHP VISCOMETER. This automated instrument (shown in Fig. 18) is a high temperature, high pressure rotational viscometer with upper operating limits of 500°F and 15,000 psig. The obtainable shear rates range from 5 to 1500 sec^{-1}. Shear stresses can be measured between 1 and 420 lb/100 ft^2. The viscometer consists of concentric cylinders mounted in an autoclave. The inner cylinder is rotated by utilization of magnetic coupling and the measurements are obtained through use of a torsion bar. The pressure is controlled by a calibrated piston which varies the volume of the cell. Therefore, density of the test fluid at the test temperatures and pressures can be calculated. Temperature, pressure and shear rates each have a manual override. Data generated by use of this instrument is gathered and analyzed by the computer.

6.2.2.2 Operating Procedures

Specific operating procedures for these instruments can be obtained from the manufacturer.

6.3 TELESCOPIC-SHEAR VISCOMETER—FANN MODEL 5STDL CONSISTOMETER

6.3.1 Description

This is a high pressure, high temperature instrument (shown in Fig. 19) in which the test fluid is subjected to telescopic shear. The upper operating limits are 20,000 psi and 500° F. Axial movement of the iron bob is caused by two alternately energized electro-magnets positioned at ends of the sample cavity. The fluid is sheared in the annular space between two coaxial cylinders, the outer forming the sample container and the moving bob being the inner member. Bob movement is retarded in proportion to the viscosity of the test fluid. The travel time is a measurement of relative viscosity.

6.3.2 Limitations

Absolute viscosity is not determined with this instrument and the results are usually considered as relative viscosity.

Table 2—High Temperature, Pressurized Concentric Cylinder Viscometers

Model	Viscosity Range cP*	Rotor Speed rpm	Maximum Temperature °F	Maximum Pressure psi
Chandler 7400	0-54,000	0-600 variable	400	20,000
Fann 50C	1-300,000	1-625 variable	500	1,000
Fann 70	1-300,000	1-625 variable	500	20,000
Fann 5STDL	N/A	N/A	500	20,000
Haake RV20/D100	1-10,000	0-1,000 variable	662	14,000
Huxley Bertram HTHP	1-200,000	0-1,000 variable	500	15,000

*Newtonian viscosity.

A constant force is imposed on the bob by the electromagnets so that it must accelerate from zero to its terminal velocity in the test fluid. In typical drilling fluids, the bob may not always travel at uniform velocity so that the analysis at a constant and defined shear rate in the annulus may not be possible.

6.3.3 Operating Procedures

Specific operating procedures for this instrument should be obtained from the manufacturer.

6.4 PIPE VISCOMETER

6.4.1 Description

Pipe viscometers are highly varied in form and intent. Essentially the instrument is a tube or pipe of sufficient length to develop fully the shear rates and type of flow of interest and a diameter that closely simulates actual field use conditions. This tube is coupled with a pumping source of sufficient size to meet desired parameters. Careful control and measurement of flow rate are necessary and usually are accomplished by using a variable speed pump and flow meters. When the desired flow rate is obtained, the pressure drop of the fluid is measured through a specified test section of the pipe. Viscosity may then be calculated from standard equations using the shear rate, pressure drop, diameter and length. The configuration of the viscometer may be altered to investigate annular flow by placing a pipe of a smaller diameter inside the pipe viscometer tube and flowing in the annulus.

6.5 PORTABLE CAPILLARY VISCOMETER

6.5.1 Description

This instrument consists basically of a fluid reservoir, heating jacket, pressure gauge, three-port valve, coiled cap-

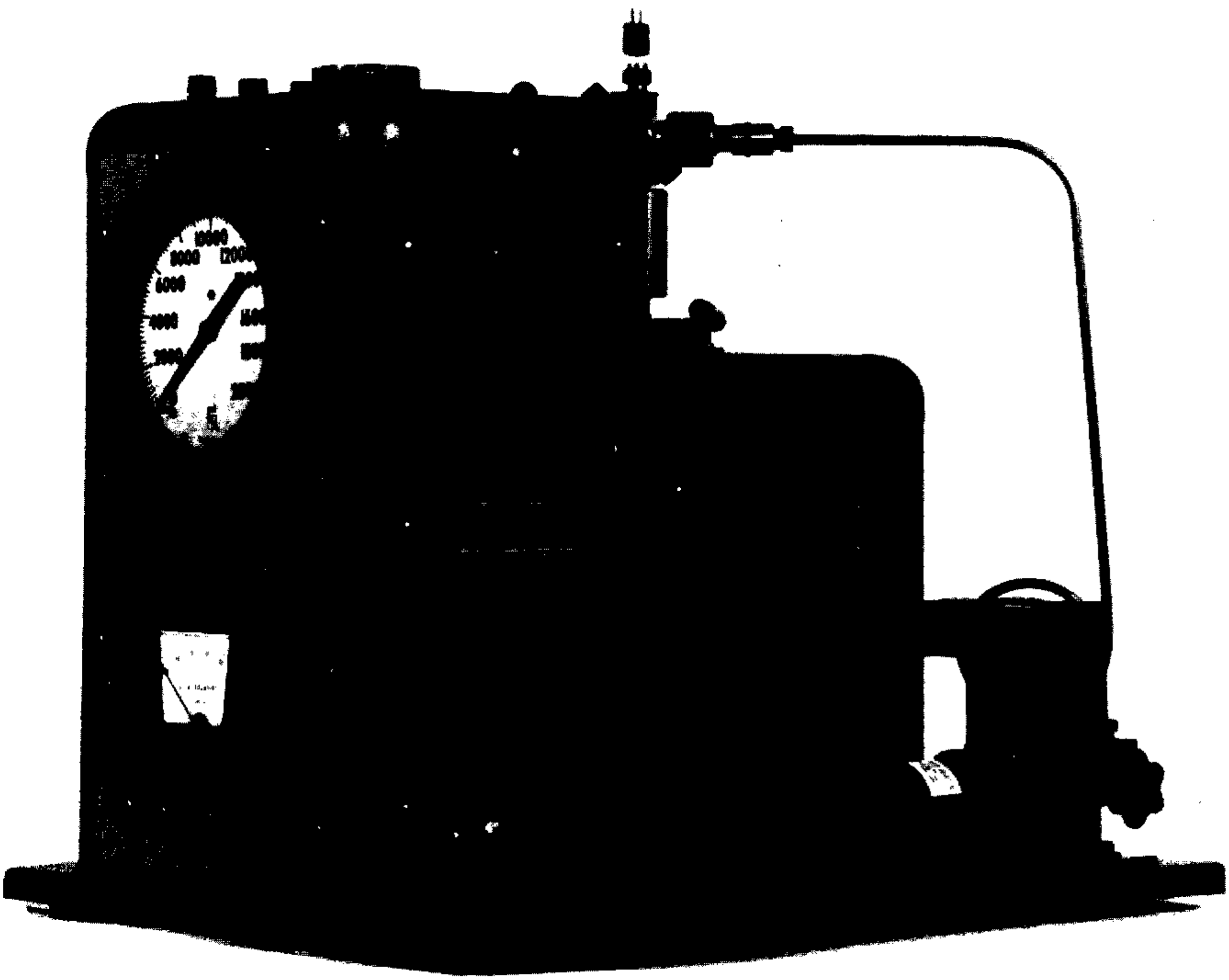

Figure 19—Fann 55TDL Consistometer

illary tube and two interchangeable straight capillary tubes. A drilling fluid sample is placed in the reservoir and pressured by gas from a portable nitrogen source. The gas forces the drilling fluid out through either the coiled capillary tube or one of the two interchangeable straight capillary tubes, depending upon the positioning of the port valve. The coiled tube is used in the low shear rate range (10-10,000 sec^{-1}). The two straight tubes are used in the higher shear rate range (1,000-100,000 sec^{-1}). No matter which tube is in use, the length is sufficient to insure that flow is fully developed before entering the test section. During each measurement, the pressure drop, indicated by the gauge, and the flow rate are recorded. The reservoir pressure is varied to cover the desired range of shear rates. The gel strength of the fluid is measured in the coiled tube. The pressure required to begin flow is measured after the drilling fluid has remained stationary for the desired gelation time.

6.5.2 Calculation

Equations for determining shear stress, shear rate and viscosity from such instruments are discussed in Sections 4 and 7.

6.5.3 Operating Procedures

Specific operating procedures for this instrument should be obtained from the manufacturer.

7 Data Analysis

7.1 DESCRIPTION

This section describes methods for analyzing drilling fluid rheological data and presents the means for estimating the effects of temperature and pressure.

7.2 RHEOLOGICAL PROFILE CURVES

Rheological data can be shown on linear, semi-log or log-log graphs of shear rate versus shear stress or viscosity. The data have also been plotted as viscometer dial readings versus viscometer rpm. In most cases, it is preferable to show the dependent variable, shear stress (τ), viscosity (μ) or viscometer dial reading on the vertical axis. Values of shear stress can be expressed in dynes/cm² or lb/100 ft². Viscosity is usually expressed as centipoise (cP). Shear rate is expressed as reciprocal second (sec^{-1}). The foregoing only applies to instruments similar to the concentric cylinder viscometer described in Section 6.

Shear stress or viscosity versus shear rate relationships are useful in classifying fluids and in the mathematical treatment of data. Fig. 20 is a linear plot and Fig. 21 is a log-log plot of several flow models.

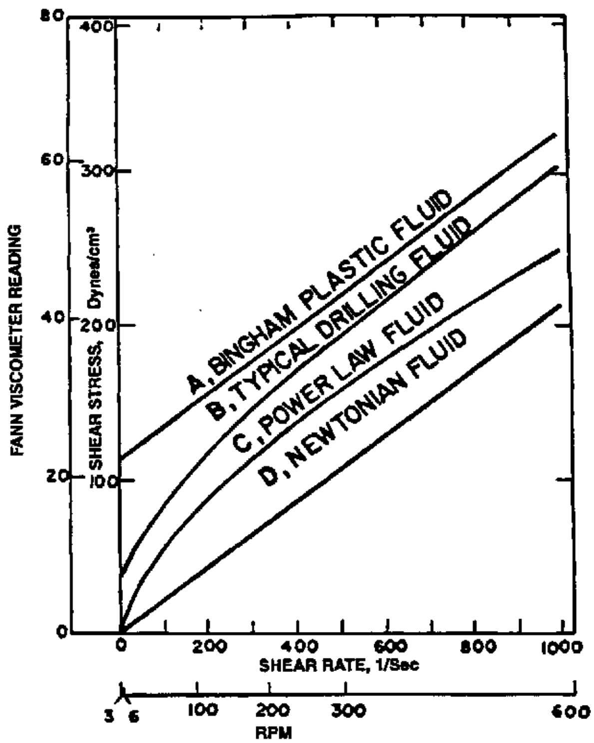

Figure 20—Linear Shear Stress—Shear Rate Plots

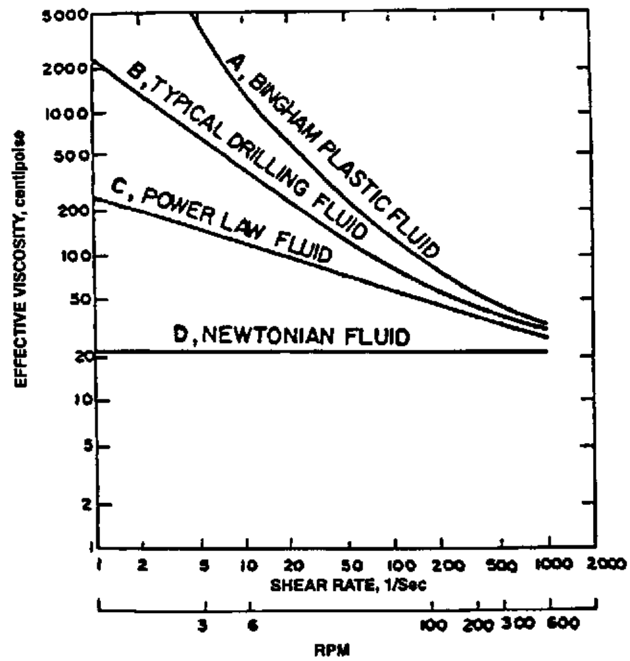

Figure 21—Log-Log Effective Viscosity—Shear Rate Plots

7.3 MATHEMATICAL FLOW MODELS

These models provide a means of using viscometer data or shear stress/shear rate relationships to develop usable information. They are a means of determining the effective viscosity as described in Par. 4.2 from which hydraulic calculations are made.

Effective viscosity is defined by the following equation:

$$\mu_e = \frac{\tau}{\gamma} \qquad (16)$$

Where:

τ = shear stress
γ = shear rate
μ_e = effective viscosity at the specified shear rate

The effective viscosity relationship obtained in this manner can be used in many of following calculations.

7.3.1 Newtonian Model

Newtonian fluids, as defined in Par. 5.1, follow a simple linear equation in laminar flow:

$$\tau = \mu\,\gamma \qquad (17)$$

When the shear stress (τ) of a Newtonian fluid is plotted against the shear rate (γ) in linear coordinates, a straight line through the origin results. The Newtonian viscosity (μ) is the slope of this line. The effective viscosity of a Newtonian fluid can be expressed as:

$$\mu_e = \frac{\tau}{\gamma} = \mu \qquad (18)$$

Since the shear stress/shear rate ratio is a constant for any shear rate, the effective viscosity is equal to the Newtonian viscosity and is independent of shear rate.

7.3.2 Non-Newtonian Model[11]

7.3.2.1 Bingham Plastic Model

A Bingham plastic fluid is one in which flow occurs only after a finite stress, known as yield stress or yield point, is applied. The stress required to initiate flow can vary from a small to a large value. After the yield stress has been exceeded, the shear stress is proportional to the shear rate.

$$\tau - \tau_y = \eta\,\gamma \qquad (19)$$

Where:

τ_y = yield point (or yield stress)
η = plastic viscosity

Analysis of Bingham plastic data can be found in Section 7.4.

7.3.2.2 Power Law Model

The power law model is:

$$\tau = K\,\gamma^n \qquad (20)$$

Where:

K = fluid consistency index
n = power law exponent

A plot of shear stress versus shear rate in linear coordinates results in a curve. It is apparent from the exponential form, however, that a plot of shear stress versus shear rate in log-log coordinates gives a straight line where n is the slope and K is the intercept at $\gamma = 1$. Logarithmic plots of effective viscosity (μ_e) versus shear rate (γ) are shown as B and C in Fig. 21. The idealized straight line plot shown as C is seldom encountered in actual practice. Plots of field drilling fluid data more nearly resemble line B. See Section 7.5 below for specific mathematical procedures that can be used to determine the power law parameters for drilling fluids.

7.3.2.3 Herschel-Buckley (Modified Power Law) Model

The Herschel-Buckley model is a three-parameter model which combines the features of the Newtonian, Bingham plastic and power law models. It allows for a yield stress followed by power law behavior at higher stress levels. The Herschel-Buckley model is:

$$\tau - \tau_y = K\,\gamma^n \qquad (21)$$

Where:

τ_y = yield stress, lb/100 ft^2

If the yield stress is equal to zero, power law behavior is described. If the flow exponent (n) is equal to 1, Bingham plastic behavior is described. If the yield stress is equal to zero and n = 1, Newtonian behavior is described and K is the Newtonian viscosity. A subsequent log-log plot of ($\tau - \tau_y$) versus γ will be similar to that of a power law plot with the slope being the exponent n and the intercept at $\gamma = 1$, the constant K.

7.3.2.4 Casson Model

The Casson model is a two-parameter model written as follows:

$$\tau^{\frac{1}{2}} = \tau_y^{\frac{1}{2}} + \mu_\infty^{\frac{1}{2}}\,\gamma^{\frac{1}{2}} \qquad (22)$$

The two parameters in the model consist of a yield stress (τ_y) and the viscosity at infinite shear rate (μ_∞). A plot of $\tau^{1/2}$ versus $\gamma^{1/2}$ in linear coordinates gives $\tau_y^{1/2}$ as the intercept and $\mu_\infty^{1/2}$ as the slope of the straight line. The Casson model often fits drilling fluid rheological data better than other two-parameter, Bingham plastic or power law models, in the low shear rate region. However, use of the Casson model for performing pressure loss calculations is a fairly complicated and difficult process and is rarely attempted.

[11]Refer to References 16, 17, 25, 29, and 30.

7.4 ANALYSIS OF BINGHAM PLASTIC MODEL DATA

7.4.1 Very few fluids actually follow the Bingham plastic model over the shear rate range of interest, but the empirical significance of the constants has become so firmly entrenched in drilling fluid technology that the yield point (τ_y), in $lb_f/100\ ft^2$, and plastic viscosity (η) in cP, are two of the best known properties of drilling fluids. They are calculated from the standard concentric cylinder viscometer (see Par. 6.2) readings at 600 rpm and 300 rpm (R_{600} and R_{300}) as follows:

$$\eta = PV = R_{600} - R_{300} \quad (23)$$

$$\tau_y = YP = R_{300} - PV \quad (24)$$

7.4.2 The average velocity of a drilling fluid in the pipe is determined by the use of the formula:

$$V_p = \frac{0.408\ Q}{D^2} \quad (25)$$

Where:

V_p = average velocity of the fluid in the pipe (ft/sec)
Q = volumetric flow rate (gal/min)
D = inner diameter of pipe (in)

7.4.3 In the annulus, the average velocity is determined by:

$$V_a = \frac{0.408\ Q}{D_2^2 - D_1^2} \quad (26)$$

Where:

V_a = average velocity of the fluid in the annulus (ft/sec)
D_1 = inner annulus diameter (in)
D_2 = outer annulus diameter (in)

7.4.4 An explicit expression for shear rate at the pipe wall as a function of velocity cannot be derived from the Bingham equation; but in a pipe of diameter (D), the effective viscosity can be approximated by:

$$\mu_e = \frac{6.65\ \tau_y\ D}{V_p} + \eta \quad (27)$$

Where:

τ_y = yield stress ($lb/100\ ft^2$)
η = plastic viscosity (cP)

7.4.5 In the annulus, the effective viscosity can be approximated by:

$$\mu_e = \frac{5.45\ \tau_y (D_2 - D_1)}{V_a} + \eta \quad (28)$$

Note: In the above equation, the constant 5.45 is true only for a D_1/D_2 ratio of 0.5 but varies only slightly from 5.49 to 5.43 over a range of diameter ratios from 0.3 to 0.9.[12]

7.5 MATHEMATICAL ANALYSIS OF POWER LAW DATA

The rheological parameters n and K can be calculated from any two shear rate-shear stress data points. Since it is rare that a log-log plot of all rheological data will be a straight line, it is better to determine n and K at the shear rates that exist inside a pipe and in an annulus. Better accuracy will result from the use of n and K in the 5 to 200 sec^{-1} shear rate range for the annulus and in the 200 to 1000 sec^{-1} shear rate range for inside pipe.

The viscometer dial readings from a standard six-speed instrument can be used to determine the power law constants. Normal practice is to use the 3-rpm and 100-rpm readings for the low shear rate range and the 300-rpm and 600-rpm reading for the high shear rate range. If a two-speed instrument is being used, the 100-rpm reading can be estimated from the 300-rpm and 600-rpm data by use of the equation:

$$R_{100} = R_{300} - \frac{2\ (R_{600} - R_{300})}{3} \quad (29)$$

Where:

R_{100} = Fann viscometer reading at 100 rpm
R_{300} = Fann viscometer reading at 300 rpm
R_{600} = Fann viscometer reading at 600 rpm

7.5.1 The general formulas for n and K are:

$$n = \frac{\log(\tau_2/\tau_1)}{\log(\gamma_2/\gamma_1)} \quad (30)$$

$$K = \frac{\tau_2}{\gamma_2^n} \quad (31)$$

Where:

n = power law exponent
K = fluid consistency index ($dyne\ sec^{-n}/cm^2$)
τ_1 = shear stress at shear rate 1
τ_2 = shear stress at shear rate 2
γ_1 = shear rate 1
γ_2 = shear rate 2

7.5.2 Using data obtained at 600 rpm and 300 rpm, the parameters to be used for inside pipe calculations are:

$$n_p = \frac{\log(R_{600}/R_{300})}{\log(1022/511)} = 3.32 \log \frac{R_{600}}{R_{300}} \quad (32)$$

$$K_p = \frac{5.11\ R_{300}}{511^{n_p}} \quad \text{or} \quad \frac{5.11\ R_{600}}{1022^{n_p}} \quad (33)$$

[12]Refer to Reference 22.

7.5.3 Using data obtained at 100 rpm and 3 rpm, the parameters to be used for inside pipe calculations are:

$$n_a = \frac{\log(R_{100}/R_3)}{\log(170.2/5.11)} = 0.657 \log(R_{100}/R_3) \quad (34)$$

$$K_a = \frac{5.11 R_{100}}{511^{n_a}} \text{ or } \frac{5.11 R_3}{5.11^{n_a}} \quad (35)$$

7.5.4 Using data obtained at 100 rpm and 3 rpm, the parameters to be used in calculating settling velocities are:

$$n_s = 0.657 \log(R_{100}/R_3) \quad (36)$$

$$K_s = \frac{5.11\, R_{100}}{170.2^{n_s}} \text{ or } \frac{5.11\, R_3}{(5.11)^{n_s}} \quad (37)$$

Where:

V_s = average settling velocity (ft/sec)
D_p = effective particle diameter (in)

7.5.5 The general power law equation for effective viscosity (cP) is:

$$\mu_e = 100\, K\, \gamma^{n-1} \quad (38)$$

7.5.6 The effective viscosity (cP) in a pipe is:

$$\mu_{e_p} = 100\, K_p \left(\frac{96 V_p}{D}\right)^{(n_p-1)} \left(\frac{3n_p + 1}{4n_p}\right)^{n_p} \quad (39)$$

7.5.7 The effective viscosity (cP) in an annulus is:

$$\mu_{e_a} = 100\, K_a \left(\frac{144 V_a}{D_2 - D_1}\right)^{(n_a-1)} \left(\frac{2n_a + 1}{3n_a}\right)^{n_a} \quad (40)$$

7.5.8 The effective viscosities μ_{ep} and μ_{ea} can be used to determine pressure losses as outlined in Section 8.

7.5.9 The effective viscosity (cP) of fluid surrounding a settling particle is:

$$\mu_{e_s} = 100\, K_s \left(\frac{12 V_s}{D_p}\right)^{(n_s-1)} \quad (41)$$

7.5.10 The effective viscosity (μ_{es}) can be used to determine settling velocities as outlined in Section 9.

7.6 EFFECTS OF TEMPERATURE AND PRESSURE ON VISCOSITY[13]

7.6.1 Temperature Effect

As the temperature increases, the effective viscosity decreases. The temperature effect[14] is described mathematically as:

$$\mu_e(T_2) = \mu_e(T_1) \exp\left[\beta\left(\frac{T_2 - T_1}{T_1 T_2}\right)\right] \quad (42)$$

Where:

$\mu_e(T_2)$ = effective viscosity at temperature 2
$\mu_e(T_1)$ = effective viscosity at temperature 1
T_1 = absolute temperature 1
T_2 = absolute temperature 2
β = temperature constant

This approximation holds until a thermal decomposition or transition point of any component of the drilling fluid is reached. Above this temperature, the fluid flow properties do not follow any mathematical model. The temperature constant, β, must be determined at each shear rate for each drilling fluid. As a general rule, the temperature effect is high for oil-based fluids containing asphalt, moderate for oil-based fluids with oil-wet inorganic solids as viscosifiers, and low for water-based fluids.

7.6.2 Pressure Effect

As the pressure increases, the effective viscosity increases. The pressure effect is described mathematically as:

$$\mu_e(P_2) = \mu_e(P_1) \exp\left[\alpha(P_2 - P_1)\right] \quad (43)$$

Where:

$\mu_e(P_2)$ = effective viscosity at pressure 2
$\mu_e(P_1)$ = effective viscosity at pressure 1
α = pressure constant
P_1 = pressure 1
P_2 = pressure 2

The pressure constant, α, must be determined for each drilling fluid. For water-based fluids, the pressure effect on shear stress is extremely small and can be neglected. However, for oil-based fluids the pressure has an appreciable effect on the effective viscosity. As a general rule, the pressure effect is greater for oil-based fluids with asphaltic viscosifiers than for those that use oil-wet inorganic solids as viscosifiers.

Note: Absolute temperature is in degrees Rankine (460 + °F). Pressure is in psig.

7.6.3 Application

The use of viscosity measurements at surface conditions for calculating hydraulics may give erroneous results.[15] For accurate work, the viscosity of the drilling fluid should be determined at the temperatures and pressures encountered in the well. To do this requires a high temperature-high pressure viscometer for data collection and a computer to analyze the data. However, corrections can be made to surface

[13]Refer to References 22 and 23.
[14]Refer to References 17 and 31.

[15]Refer to Reference 29.

conditions. These correction factors are average values obtained from measurements on various types of drilling fluids under conditions of high temperature and high pressure. Although the use of these correction factors will give good estimates, they are not as accurate as downhole viscosities that can be obtained by measurement under downhole conditions. Figs. 22, 23, and 24 show the correction factor to be used with water-based fluids, oil-based fluids containing asphalt, and oil-based fluids containing oil-wet inorganic viscosifiers, respectively. To obtain the correction factor:

a. Select the proper graph to be used.

b. At the temperature of interest, draw a line to the proper pressure curve.

c. From the intersection of the temperature-pressure lines, draw a line to the correction factor axis and read the correction factor.

d. Multiply the effective viscosity by the correction factor.

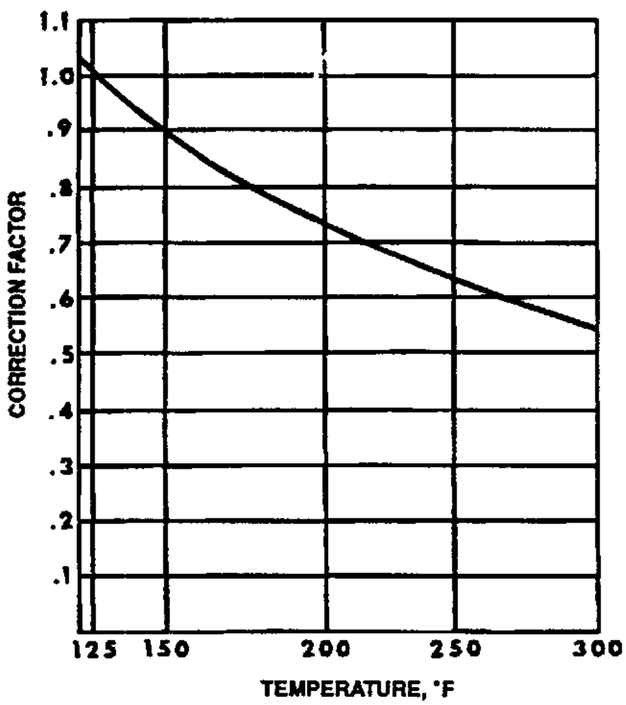

Figure 22—Downhole Viscosity Correction Factor
Water Base Mud

8 Application of Rheological Data

8.1 DESCRIPTION

Rheological data are used to determine drilling fluid hydraulics. The calculations shown in this section have been simplified; however, the results obtained are sufficiently accurate for field operations.

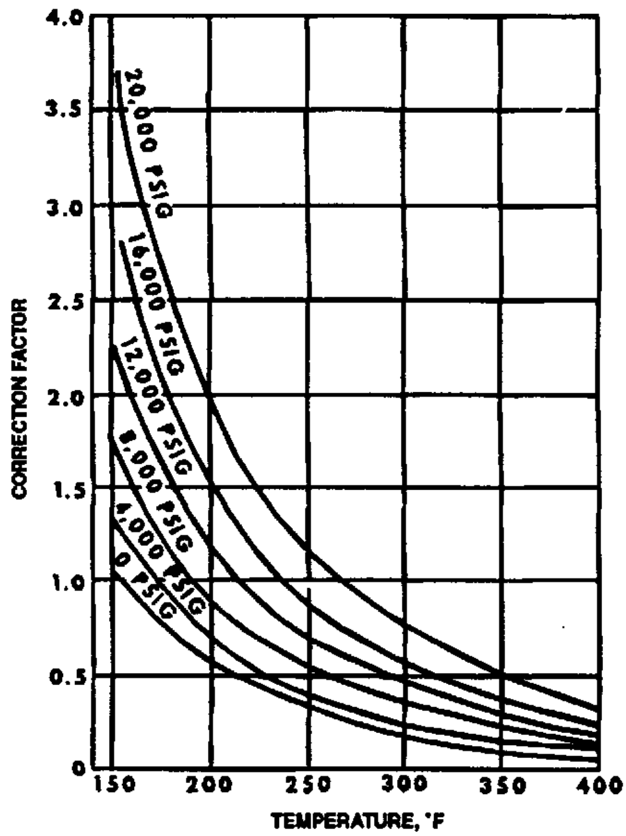

Figure 23—Downhole Viscosity Correction Factor
Oil Muds Containing Asphalt

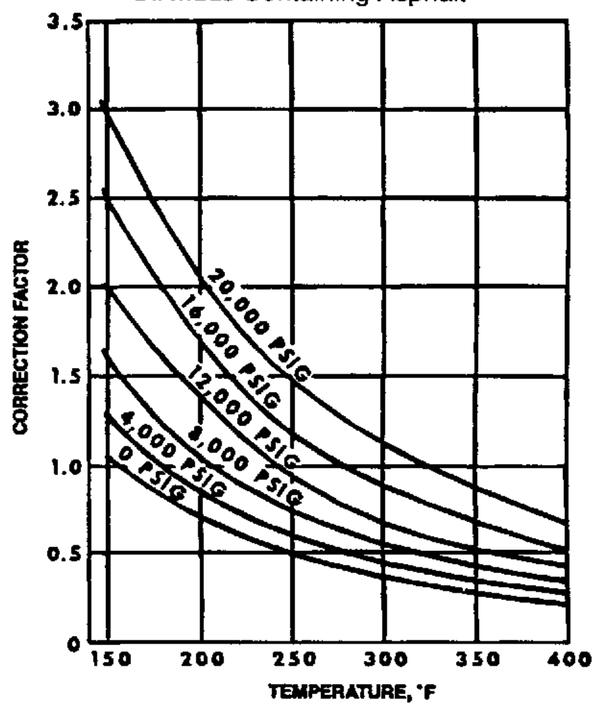

Figure 24—Downhole Viscosity Correction Factor
Oil Muds Containing Oil-Wet Inorganic Viscosifiers

8.2 FRICTION LOSS IN PIPE

8.2.1 Calculation of Reynolds Number[16]

After obtaining the effective viscosity (μ_{ep}) as a function of the operating shear rate at the pipe wall (γ_p), the reynolds number in the pipe (N_{Rep}) is calculated from:

$$N_{Re_p} = \frac{928\, V_p\, D\, \rho}{\mu_{ep}} \tag{44}$$

Note: μ_{ep} can be calculated according to Eq. 39.

8.2.2 CALCULATION OF THE FRICTION FACTOR

a. If the Reynolds Number is less than or equal to 2100, the friction factor in the pipe is:

$$f_p = \frac{16}{N_{Re_p}} \tag{45}$$

b. If the Reynolds Number is greater than 2100, the friction factor can be estimated from:

$$f_p = \frac{a}{(N_{Re_p})^b} \tag{46}$$

Where:

a = (log n + 3.93)/50
b = (1.75 – log n)/7

8.2.3 Calculation of Friction Loss Pressure Gradient in Drill Pipe

The appropriate friction factor, which is dimensionless, is then substituted into the Fanning equation to obtain the friction loss pressure gradient.

$$P_p/L_m = \frac{f_p\, V_p^2 \rho}{25.81\, D} \tag{47}$$

Where:

L_m = Length of drill pipe (ft)

Note: Reynolds Number and friction loss must be calculated for each section of pipe having different inside diameters.

8.3 FRICTION LOSS IN AN ANNULUS

8.3.1 Calculation of Reynolds Number

The Reynolds Number in the annulus is calculated from the following equation:

$$N_{Re_p} = \frac{928\, V_a\, (D_2 - D_1)\, \rho}{\mu_{ea}} \tag{48}$$

Note: μ_{ea} can be calculated according to Eq. 40.

[16]Refer to References 14, 19, and 27.

8.3.2 Calculation of the Friction Factor

a. If the Reynolds Number is less than or equal to 2100, the friction factor in the pipe is

$$f_a = \frac{24}{N_{Re_a}} \tag{49}$$

b. If the Reynolds Number is greater than 2100, the friction factor can be estimated from:

$$f_a = \frac{a}{(N_{Re_a})^b} \tag{50}$$

Where:

a = (log n + 3.93)/50
b = (1.75 - log n)/7

8.3.3 Calculation of the Friction Loss Pressure Gradient

The appropriate friction factor is then substituted in the Fanning equation for an annulus to obtain the friction loss pressure gradient (P_a/L) in lb/in²/ft:

$$P_a/L_m = \frac{f_a\, V_a^2\, \rho}{25.81\, (D_2 - D_1)} \tag{51}$$

Note: Reynolds Number and friction loss must be calculated for each section of the annulus having different annular diameters.

8.3.4 Average Friction Loss Pressure Gradient

If more than one section of annulus is present, an average friction loss pressure gradient for the well is calculated by use of the following equation:

$$Ave\ P_a/L_m = \frac{(P_{a1}/L_1)\, L_1 + (P_{a2}/L_2)\, L_2 + \ldots}{L_m} \tag{52}$$

8.4 FRICTION LOSS IN BIT NOZZLES

The friction loss (P_n) in bit nozzles (assuming a nozzle efficiency of 0.95) in lb/in² is calculated by use of the equation:[17]

$$P_n = \frac{156\, \rho\, Q^2}{(D_{n1}^{\,2} + D_{n2}^{\,2} + \ldots)} \tag{53}$$

Where:

ρ = mud density (lb/gal)
Q = volumetric flow rate (gal/min)
D_n = diameter of bit nozzles (¹/₃₂ inch)

[17]Refer to Reference 28.

8.5 HYDROSTATIC PRESSURE GRADIENT

The hydrostatic pressure gradient (P_h/L_v) in lb/in²/ft can be obtained from the equation:

$$P_h / L_v = 0.052\,\rho \qquad (54)$$

Where:

L_v = true vertical depth (ft)

8.6 CIRCULATING PRESSURE GRADIENT

The hydrostatic pressure gradient plus the friction loss pressure gradient in the annulus gives the circulating pressure gradient (P_c/L) in the annulus. This can be calculated as follows:

$$P_c/L = P_h/L_v + P_a/L_m \qquad (55)$$

Note: If more than one annular section is present, use the average friction loss pressure gradient in the annulus (Ave P_a/L_m) to calculate the circulating pressure gradient.

8.7 EQUIVALENT CIRCULATING DENSITY

The equivalent circulating density (ρ_c) in lb/in²/ft is calculated by use of the equation:

$$\rho_c = \frac{19.265\,P_c}{L_v} \qquad (56)$$

8.8 STANDPIPE PRESSURE

The total pressure required to circulate the fluid down the drill string, through the bit and back to the surface is the sum of all pressure losses in the circulating system.

$$P_{sp} = \sum\left(\frac{P_{pi}}{L_{pi}}\right) L_{pi} + \sum\left(\frac{P_{aj}}{L_{aj}}\right) L_{aj} + P_n \qquad (57)$$

Where:

P_{sp} = standpipe pressure (lb/in²)

The calculated standpipe pressure should be comparable to that measured on the rig.

9 Settling Velocity of Drill Cuttings

9.1 DESCRIPTION

9.1.1 Settling velocity (slip velocity) refers to the velocity at which a particle falls in a fluid. The factors controlling the settling velocity are: the size and shape of the particle, the density of the particle and the density and rheological properties of the fluid through which the particle settles.[18]

9.1.2 Calculations of settling velocities, as outlined in this section, pertain only to vertical or near vertical boreholes.

[18]Refer to References 4, 9, 10, and 26.

9.2 SETTLING OF PARTICLES IN WATER

9.2.1 Drilled cuttings are irregularly shaped particles. The equivalent diameter of an irregularly shaped particle can be determined from its volume according to:

$$D_p = \sqrt[3]{\frac{6\,V_o}{\pi}} \qquad (58)$$

Where:

V_o = volume of particle, in³

The volume of a particle can be determined from its dimensions or its submerged volume. Either a nominal or an equivalent diameter is used to describe particle size. Since settling velocity calculations are based on settling of spheres, a correction factor must be applied to account for the geometry of irregular shaped particle. Table 3 provides an estimate of the equivalent spherical diameter for irregularly shaped particles.[19]

Table 3—Equivalent Diameters of Irregularly Shaped Particles

Volume, in³	Equivalent Fraction Diameter, in.	Equivalent Decimal Diameter, in.
0.0010	1/8	0.125
0.0082	1/4	0.250
0.0276	3/8	0.375
0.0650	1/2	0.500
0.1280	5/8	0.625
0.2210	3/4	0.750
0.3510	7/8	0.875
0.5230	1	1.000
0.7460	1 1/8	1.125
1.2230	1 1/4	1.250
1.3610	1 3/8	1.375
1.7670	1 1/2	1.500

EXAMPLE:

Suppose a particle has the following dimensions:

Length:	1 inch
Width:	1/2 inch
Thickness:	1/4 inch

The volume of the particle is 0.125 in.³ Referring to Table 3, the equivalent diameter is 0.625, or 5/8 inch.

9.2.2 The settling velocity of various sized particles in water is shown in Fig. 25. This log-log plot distinctly shows that for particles of the same density, the settling velocity increases directly with the particle size.[20]

[19]Refer to Reference 15.
[20]Refer to Reference 11.

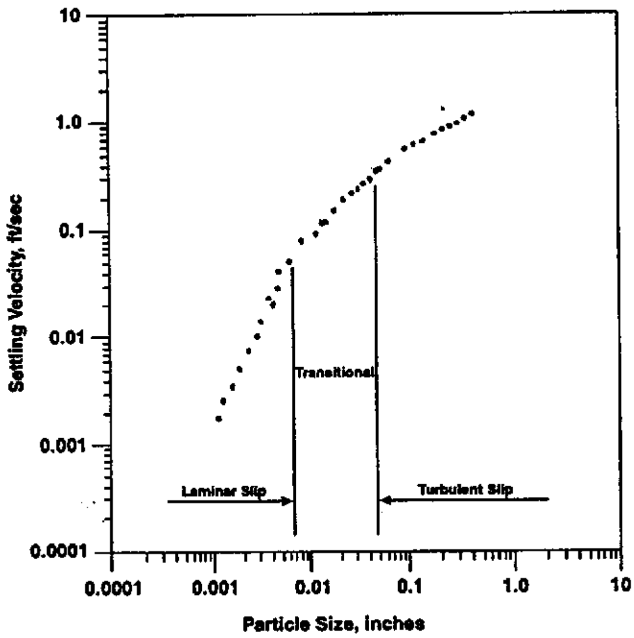

Figure 25—Settling Velocity of Drill Cuttings in Water

9.2.3 There are three different slip regimes which control the settling velocity—laminar, transitional, and turbulent slip.

a. In laminar slip, the settling velocity increases with the square of the particle diameter. The viscosity of the fluid through which the particle settles has a dominant effect. This is known as Stokes' Law.

b. In turbulent slip, the settling velocity is proportional to the square root of the particle size and the density of the fluid has a dominant role.

c. Transitional slip is the region between laminar and turbulent slip. Both density and viscosity are important in describing settling in transitional slip.

9.3 ESTIMATION OF SETTLING VELOCITY

9.3.1 Settling velocities may be estimated by use of the correlation:[21]

$$V_s = 0.0002403e^{5.030\Psi}\left(\frac{\mu_{e_s}}{D_p\rho}\right)\left(\sqrt{1+(920790.49e^{-5.030\Psi})D_p\left(\frac{\rho_p}{\rho}-1\right)\left(\frac{D_p\rho}{\mu_{e_s}}\right)^2}-1\right) \quad (59)$$

Where:

V_s = settling velocity, ft/sec
Ψ = (surface area of sphere with same volume as particle) ÷ (surface area of particle)
μ_{es} = effective viscosity of non-Newtonian fluids in settling, cP
D_p = equivalent diameter of particle, inch
ρ = density of fluid, lb/gal
ρ_p = density of particle, lb/gal

9.3.2 For most commonly encountered irregular particles, the value of Ψ is approximately 0.8 and Eq. 59 is simplified to:

$$V_S = 0.01294\left(\frac{\mu_{e_s}}{D_p\rho}\right)\left(\sqrt{1+17106.35D_p\left(\frac{\rho_p}{\rho}-1\right)\left(\frac{D_p\rho}{\mu_{e_s}}\right)^2}-1\right) \quad (60)$$

9.3.3 For Newtonian fluids, the viscosity is independent of the shear rate and the effective viscosity is the same as the Newtonian viscosity. The settling velocity can be estimated by a single calculation.

9.3.4 For non-Newtonian fluids, the effective viscosity depends on the shear rate. The viscosity can be calculated by use of the power law models shown in Section 5. Since the shear rate is determined by the settling velocity, a numerical iteration method must be used to estimate the settling velocities for non-Newtonian fluids.

[21]Refer to Reference 12.

APPENDIX A—RHEOLOGICAL EXAMPLE CALCULATIONS

A.1 Well Information

a. Flow rate, Q = 280 gal/min
b. Drilling fluid density, ρ = 12.5 lb/gal
c. Drill pipe
  1. Length, L = 11,400 ft
  2. Outside diameter, D_1 = 4.5 in.
  3. Inside diameter, D = 3.78 in.
d. Drill collars
  1. Length, L = 600 ft
  2. Outside diameter, D_1 = 6.5 in.
  3. Inside diameter, D = 2.5 in.
e. Surface casing
  1. Length, L = 3,000 ft
  2. Inside diameter, D_2 = 8.835 in.
f. Bit
  1. Diameter, D_2 = 8.5 in.
  2. Nozzles = 11, 11, 12 1/32 in.
g. Drilling fluid viscosity
  1. Fann Viscometer reading at 600 rpm
    a. $\tau = 65$ lb/100 ft^2
    b. $\gamma = 1022$ sec^{-1}
  2. Fann Viscometer reading at 300 rpm
    a. $\tau = 39$ lb/100 ft^2
    b. $\gamma = 511$ sec^{-1}
  3. Fann Viscometer reading at 100 rpm
    a. $\tau = 20$ lb/100 ft^2
    b. $\gamma = 170.2$ sec^{-1}
  4. Fann Viscometer reading at 3 rpm
    a. $\tau = 3$ lb/100 ft^2
    b. $\gamma = 5.11$ sec^{-1}

A.2 Power Law Constants (n)

a. Drill pipe

$$n_p = 3.32 \log (R_{600}/R_{300}) \quad \text{(A-1)}$$
$$= 3.32 \log (65/39)$$
$$= 0.737$$

b. Annulus

$$n_a = 0.657 \log (R_{100}/R_3) \quad \text{(A-2)}$$
$$= 0.657 \log (20/3)$$
$$= 0.541$$

A.3 Fluid Consistency Index (K)

a. Drill pipe

$$K_p = 5.11\, R_{600}/(1022)^{np} \quad \text{(A-3)}$$
$$= 5.11\, (65)/(1022)^{0.737}$$
$$= 2.017 \text{ dyne sec}^{-n}/\text{cm}^2$$

b. Annulus

$$K_a = 5.11\, R_{100}/(170.2)^{na} \quad \text{(A-4)}$$
$$= 5.11\, (20)/(170.2)^{0.541}$$
$$= 6.346 \text{ dyne sec}^{-n}/\text{cm}^2$$

A.4 Average Bulk Velocity in a Pipe (V_p)

$$V_p = \frac{0.408\, Q}{D^2} \quad \text{(A-5)}$$

a. Drill pipe

$$V_p = \frac{(0.408)(280)}{(3.78)^2} = 8.00 \text{ ft/sec} \quad \text{(A-6)}$$

b. Drill collars

$$V_p = \frac{(0.408)(280)}{(2.50)^2} = 18.28 \text{ ft/sec} \quad \text{(A-7)}$$

A.5 Average Bulk Velocity in an Annulus (V_a)

$$V_a = \frac{0.408\, Q}{(D_2^2 - D_1^2)} \quad \text{(A-8)}$$

a. Annulus section 1

$$V_a = \frac{(0.408)(280)}{(8.835)^2 - (4.5)^2} = 1.98 \text{ ft/sec} \quad \text{(A-9)}$$

b. Annulus section 2

$$V_a = \frac{(0.408)(280)}{(8.5)^2 - (4.5)^2} = 2.20 \text{ ft/sec} \quad \text{(A-10)}$$

c. Annulus section 3

$$V_a = \frac{(0.408)(280)}{(8.5)^2 - (6.5)^2} = 3.81 \text{ ft/sec} \quad \text{(A-11)}$$

A.6 Effective Viscosity in a Pipe (μ_{ep})

$$\mu_{ep} = 100\, K_p \left(\frac{96\, V_p}{D}\right)^{n_p-1} \left(\frac{3n_p+1}{4n_p}\right)^{n} \quad \text{(A-12)}$$

a. Drill pipe

$$\mu_{ep} = 100(2.017) \left(\frac{(96)(8.00)}{3.78}\right)^{0.737-1} \left(\frac{3(0.737)+1}{4(0.737)}\right)^{0.737} = 53\ cP \quad \text{(A-13)}$$

b. Drill collars

$$\mu_{ep} = 100(6.814) \left(\frac{(96)(18.28)}{3.78}\right)^{0.737-1} \left(\frac{3(0.737)+1}{4(0.737)}\right)^{0.737} = 38\ cP \quad \text{(A-14)}$$

A.7 Effective Viscosity in an Annulus (μ_{ea})

$$\mu_{ea} = 100\,K_a\left(\frac{144V_a}{(D_2-D_1)}\right)^{n_a-1}\left(\frac{2n_a+1}{3n_a}\right)^{n_a} \quad \text{(A-15)}$$

a. Annulus section 1

$$\mu_{ea} = 100(6.346)\left(\frac{(144)(1.98)}{8.835-4.5}\right)^{0.541-1}\left(\frac{2(0.541)+1}{3(0.541)}\right)^{0.541} = 106\ cP \quad \text{(A-16)}$$

b. Annulus section 2

$$\mu_{ea} = 100(6.184)\left(\frac{(144)(2.20)}{8.5-4.5}\right)^{0.541-1}\left(\frac{2(0.541)+1}{3(0.541)}\right)^{0.541} = 98\ cP \quad \text{(A-17)}$$

c. Annulus section 3

$$\mu_{ea} = 100(6.184)\left(\frac{(144)(3.81)}{8.5-6.5}\right)^{0.541-1}\left(\frac{2(0.541)+1}{3(0.541)}\right)^{0.541} = 55\ cP \quad \text{(A-18)}$$

A.8 Reynolds Number in Pipe (N_{Rep})

$$N_{Rep} = \frac{928\,D\,V_p\rho}{\mu_{ep}} \quad \text{(A-19)}$$

a. Drill pipe

$$N_{Rep} = \frac{(928)\,(3.78)\,(8)\,(12.5)}{53} = 6{,}616 \quad \text{(A-20)}$$

b. Drill collar

$$N_{Rep} = \frac{(928)\,(2.5)\,(18.28)\,(12.5)}{38} = 13{,}870 \quad \text{(A-21)}$$

A.9 Reynolds Number in Annulus (N_{Rea})

$$N_{Rea} = \frac{(928)\,(D_2-D_1)\,V_a\rho}{\mu_{ea}} \quad \text{(A-22)}$$

a. Annulus section 1

$$N_{Rea} = \frac{(928)\,(4.335)\,(1.98)\,(12.5)}{106} = 937 \quad \text{(A-23)}$$

b. Annulus section 2

$$N_{Rea} = \frac{(928)\,(4)\,(2.20)\,(12.5)}{98} = 1046 \quad \text{(A-24)}$$

c. Annulus section 3

$$N_{Rea} = \frac{(928)\,(2)\,(3.81)\,(12.5)}{55} = 1602 \quad \text{(A-25)}$$

A.10 Friction Factor in the Pipe (f_p)

The Reynolds Number is >2100

$$f_p = \frac{a}{N_{Rep}^b} \quad \text{(A-26)}$$

$$a = \frac{\log n_p + 3.93}{50} \quad \text{(A-27)}$$

$$b = \frac{1.75 + \log n_p}{7} \quad \text{(A.28)}$$

a. Drill pipe

$$f_p = \frac{0.0759}{6616^{0.3211}} = 0.0010 \quad \text{(A-29)}$$

b. Drill collar

$$f_p = \frac{0.0759}{13869^{0.2311}} = 0.0084 \quad \text{(A-30)}$$

A.11 Friction Factor in the Annulus (f_a)

The Reynolds Number is < 2100

$$f_a = \frac{24}{N_{Rea}} \quad \text{(A-31)}$$

a. Annulus section 1

$$f_a = \frac{24}{937} = 0.0256 \quad \text{(A-32)}$$

b. Annulus section 2

$$f_a = \frac{24}{1046} = 0.0230 \quad \text{(A-33)}$$

c. Annulus section 3

$$f_a = \frac{24}{1602} = 0.0150 \quad \text{(A-34)}$$

A.12 Friction Loss Pressure Gradient in the Pipe (P_p/L_m)

$$\frac{P_p}{L_m} = \frac{f_p\,V_p^2\rho}{25.81\,D} \quad \text{(A-35)}$$

a. Drill pipe

$$\frac{P_p}{L_m} = \frac{(0.0100)(8)^2(12.5)}{(25.81)(3.78)} = 0.0815\ \text{lb/in}^2\text{/ft} \quad \text{(A-36)}$$

Since the length of drill pipe is 11,400 ft, the friction loss in the drill pipe is:

$$(P_p/L_m)(L_m) = (0.0815)(11{,}400) = 929\ \text{lb/in}^2 \quad \text{(A-37)}$$

b. Drill collars

$$P_p/L_m = \frac{(0.0084)(18.28)^2(12.5)}{(25.81)(2.25)} = 0.5428\ \text{lb/in}^2\text{/ft} \quad \text{(A-38)}$$

Since the length of drill collars is 600 ft, the friction loss in the drill pipe is:

$$(P_p/L_m)(L_m) = (0.5428)(600) = 325 \text{ lb/in}^2 \quad \text{(A-39)}$$

c. Total friction loss in the drill collars is the sum of friction losses in the drill pipe and drill collars.

$$P_p = 929 + 325 + 1254 \text{ lb/in}^2 \quad \text{(A-40)}$$

A.13 Friction Loss Pressure Gradient in the Annulus (P_a/L_m)

$$\frac{P_a}{L_m} = \frac{f_a V_a^2 \rho}{25.81\,(D_2 - D_1)} \quad \text{(A-41)}$$

a. Annulus section 1

$$P_a/L_m = \frac{(0.0256)(1.98)^2\,(12.5)}{25.81\,(8.835 - 4.5)} = 0.0112 \text{ lb/in}^2\text{/ft} \quad \text{(A-42)}$$

The length of the annulus section 1 is 3000 ft. Therefore, the friction loss is:

$$(P_a/L_m)\,(L_m) = (0.0112)\,(3000) = 34 \text{ lb/in}^2 \quad \text{(A-43)}$$

b. Annulus section 2

$$P_a/L_m = \frac{(0.0230)(2.20)^2\,(12.5)}{25.81(8.5 - 4.5)} = 0.0134 \text{ lb/in}^2\text{/ft} \quad \text{(A-44)}$$

The length of the annulus section 2 is 8400 ft. Therefore, the friction loss is:

$$(P_a/L_m)(L_m) = (0.0134)(8400) = 113 \text{ lb/in}^2 \quad \text{(A-45)}$$

c. Annulus section 3

$$P_a/L_m = \frac{(0.0161)(3.81)^2\,(12.5)}{25.81\,(8.5 - 6.5)} = 0.0527 \text{ lb/in}^2\text{/ft} \quad \text{(A-46)}$$

The length of the annulus section 3 is 600 ft. Therefore, the friction loss is:

$$(P_a/L_m)(L_m) = (0.0527)(600) = 32 \text{ lb/in}^2 \quad \text{(A-47)}$$

d. Total friction loss in the annulus is the sum of friction losses in the three sections.

$$P_a = 34 + 113 + 32 = 179 \text{ lb/in}^2 \quad \text{(A-48)}$$

e. The friction loss pressure gradient for the entire annulus is the total friction loss divided by the total depth:

$$P_a/L_m = 179/12{,}000 = 0.0149 \text{ lb/in}^2\text{/ft} \quad \text{(A-49)}$$

A.14 Friction Loss in the Bit Nozzles (P_n)

$$P_n = \frac{156\,\rho\,Q^2}{(D_{n1}^2 + D_{n2}^2 + D_{n3}^2)^2} \quad \text{(A-50)}$$

$$P_n = \frac{(156)(12.5)(280)^2}{[(121) + (121) + (144)]^2} = 1026 \text{ lb/in}^2 \quad \text{(A-51)}$$

A.15 Hydrostatic Pressure Gradient (P_h/L)

$$P_h/L = 0.052\,\rho \quad \text{(A-52)}$$

$$P_h/L = 0.052\,(12.5) = 0.65 \text{ lb/in}^2\text{/ft} \quad \text{(A-53)}$$

A.16 Circulating Pressure Gradient (P_c/L)

$$P_c/L = P_h/L + P_a/L \quad \text{(A-54)}$$

$$P_c/L = 0.65 + 0.0149 = 0.6649 \text{ lb/in}^2\text{/ft} \quad \text{(A-55)}$$

A.17 Equivalent Circulating Density (ρ_c)

$$\rho_c = 19.265\,(P_c/L) \quad \text{(A-56)}$$

$$\rho_c = 19.265(0.6649) = 12.81 \text{ lb/gal} \quad \text{(A-57)}$$

APPENDIX B—SETTLING VELOCITY EXAMPLE CALCULATIONS

B.1 Well Information

a. Particle equivalent diameter, $D_p = 0.5$ in.
b. Particle density, $\rho_p = 22.5$ lb/gal
c. Mud density, $\rho = 12.5$ lb/gal
d. Mud viscosity
  1. Fann viscometer reading at 100 rpm
     a. $\tau = 20$ lb/100 ft^2
     b. $\gamma = 170.2$ sec^{-1}
  2. Fann viscometer reading at 3 rpm
     a. $\tau = 3$ lb/100 ft^2
     b. $\gamma = 5.11$ sec^{-1}

B.2 Power Law Constants (n_s)

$$n_s = 0.657 \log (R_{100}/R_3) = 0.657 \log (20/3) = 0.541 \quad \text{(B-1)}$$

B.3 Fluid Consistency Index (K_s)

$$K_s = 5.11\, R_{100}/(170.2)^{ns} = 5.11\,(20)/(170.2)^{0.541} = 6.346 \quad \text{(B-2)}$$

B.4 Initial Settling Shear Rate Estimate (γ_s)

Assume: $V_s = 1$ ft/sec

$$\gamma_s = 12\, V_s/D_p \quad \text{(B-3)}$$

$$\gamma_s = 12(1)/0.5 = 24 \text{ sec}^{-1}$$

B.5 Effective Viscosity (μ_{es})

$$\mu_{es} = 100\, K_s\, \gamma_s^{(ns-1)} = 100\,(6.346)\,(24)^{(0.541-1)} = 148 \text{ cP} \quad \text{(B-4)}$$

B.6 Settling Velocity First Approximation (V_s)

$$V_s = 0.01294 \left(\frac{\mu_e}{D_p\,\rho}\right) \left(\sqrt{1 + 17106.53\, D_p \left(\frac{\rho_p}{\rho} - 1\right) \left(\frac{D_p\rho}{\mu_e}\right)^2} - 1\right) \quad \text{(B-5)}$$

$$V_s = 0.01294 \left(\frac{148}{0.5(12.5)}\right) \left(\sqrt{1 + 17106.53\,(0.5) \left(\frac{22.5}{12.5} - 1\right) \left(\frac{(0.5)(12.5)}{148}\right)^2} - 1\right) \quad \text{(B-6)}$$

$$V_s = 0.808 \text{ ft/sec}$$

B.7 Second Settling Shear Rate Estimate (γ_s)

$$\gamma_s = 12\,(0.808)/0.5 = 19.4 \text{ sec}^{-1} \quad \text{(B-7)}$$

B.8 Effective Viscosity (μ_{es})

$$\mu_{es} = 100(6.346)(19.4)^{(0.541-1)} = 163 \text{ cP} \quad \text{(B-8)}$$

B.9 Settling Velocity Second Approximation (V_s)

$$V_s = 0.01294 \left(\frac{163}{0.5(12.5)}\right) \left(\sqrt{1 + 17106.53\,(0.5) \left(\frac{22.5}{12.5} - 1\right) \left(\frac{(0.5)(12.5)}{163}\right)^2} - 1\right) \quad \text{(B-9)}$$

$$V_s = 0.785 \text{ ft/sec}$$

B.10 Third Settling Shear Rate Estimate (γ_s)

$$\gamma_s = 12\,(0.785)/0.5 = 18.8 \text{ sec}^{-1} \quad \text{(B-10)}$$

B.11 Effective Viscosity (μ_{es})

$$\mu_{es} = 100\,(6.346)(18.8)^{(0.541-1)} = 165 \text{ cP} \quad \text{(B-11)}$$

B.12 Settling Velocity Third Approximation (V_s)

$$V_s = 0.01294 \left(\frac{165}{0.5(12.5)}\right) \left(\sqrt{1 + 17106.53\,(0.5) \left(\frac{22.5}{12.5} - 1\right) \left(\frac{(0.5)(12.5)}{165}\right)^2} - 1\right) \quad \text{(B-12)}$$

$$V_s = 0.782 \text{ ft/sec}$$

This numerical iteration method is repeated until the settling velocities of two successive calculations are equal. In the example in this Appendix, the third and fourth approximations are equal. The calculated settling velocity is 0.782 ft/sec.

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